

# PLACE & SPACE IN MULTILEVEL MODELS

TOWARDS MORE USEFUL GEOGRAPHY  
IN MULTILEVEL MODELS

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**THE GEOGRAPHIC DICHOTOMY**

**DEFINING A CLASSIC MODEL**

**IS PLACE JUST ABOUT GROUP?**

**DOES IT REALLY MATTER?**

# **THE GEOGRAPHIC DICHOTOMY**

a core tension in geographic analysis

**DEFINING A CLASSIC MODEL**

**IS PLACE JUST ABOUT GROUP?**

**DOES IT REALLY MATTER?**

**SPACE**

**PLACE**

# SPACE

# PLACE

Understanding the New Human Dynamics in  
Smart Spaces and Places: Toward a  
**Spatial** Framework

Shih-Lung Shaw<sup>\*</sup> and Daniel Sui<sup>†</sup>

<sup>\*</sup>*Department of Geography, University of Tennessee*

<sup>†</sup>*Department of Geosciences, University of Arkansas*

**SPACE**

**PLACE**

# SPACE

*The geographic system over which objects of study are related.*

- *Earth Surface*
- *Road Systems*
- *Social Networks*
- *Economic Relations*

# PLACE

*Geographic entities that are constructed by distinctiveness.*

- *Regions*
- *Neighborhoods*
- *Home/Staying locales*
- *Functional classifications*

# SPACE

*The geographic system over which objects of study are related.*

- *Earth Surface*
- *Road Systems*
- *Social Networks*
- *Economic Relations*

# PLACE

***Geographic entities that are constructed by distinctiveness.***

*How or why do they emerge?*

*What are their properties?*

*What are their purpose?*

*Do they have effects on things we care about?*

# SPACE

***The geographic system over which objects of study are related.***

*How do things interact?*

*Over what spatial systems?*

*In what manner?*

*What impact do entities have on others nearby?*

# PLACE

***Geographic entities that are constructed by distinctiveness.***

*How or why do they emerge?*

*What are their properties?*

*What are their purpose?*

*Do they have effects on things we care about?*

# SPACE

***The geographic system over which objects of study are related.***

*Dependence:*

*Nearby things are more related than distant things*

# PLACE

***Geographic entities that are constructed by distinctiveness.***

*Dependence:*

*Things in the same place are more related than things in different places*

# THE GEOGRAPHIC DICHOTOMY

space & place are core to geography

## **DEFINING A CLASSIC MODEL**

IS PLACE JUST ABOUT GROUP?

DOES IT REALLY MATTER?

# THE GEOGRAPHIC DICHOTOMY

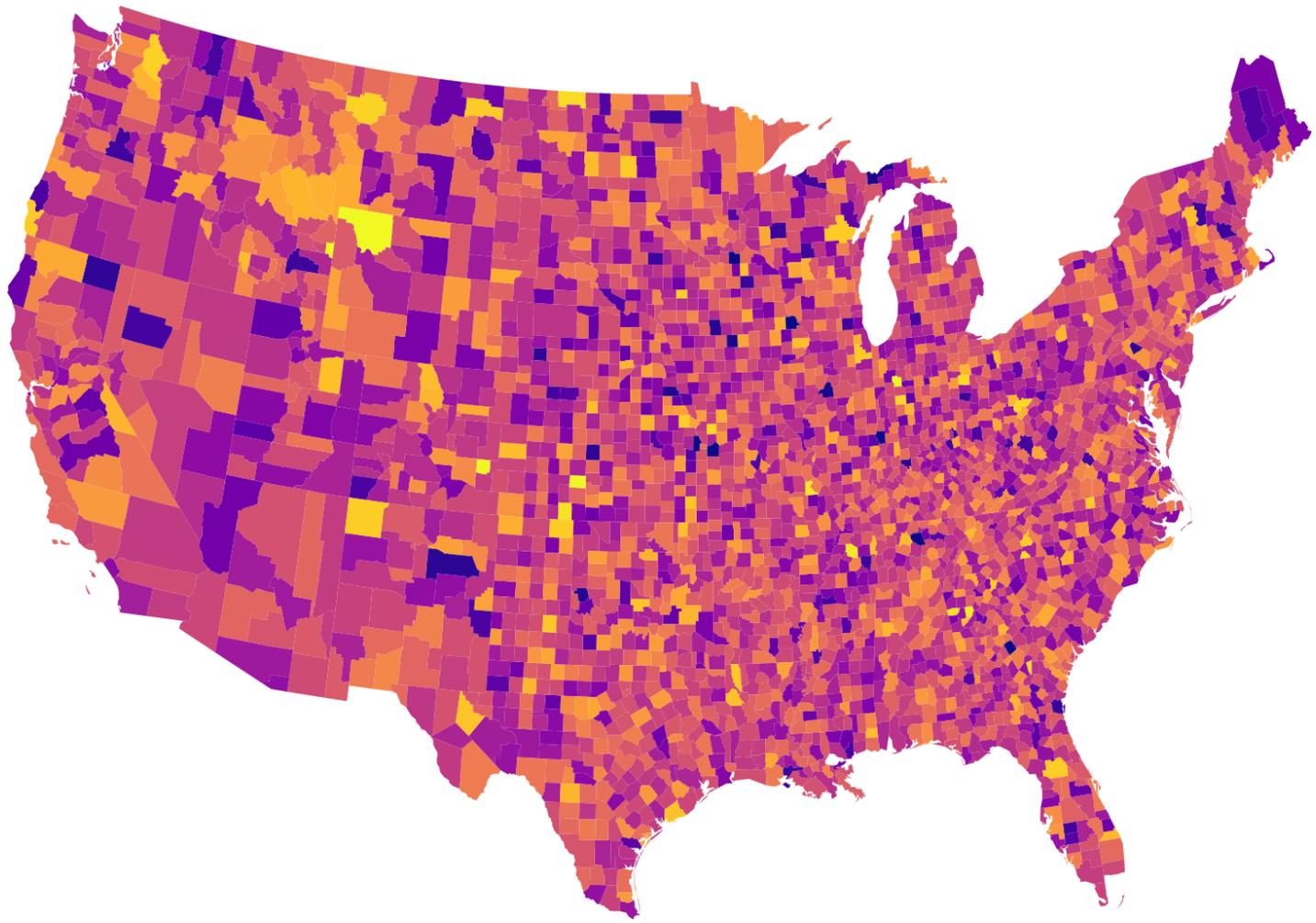
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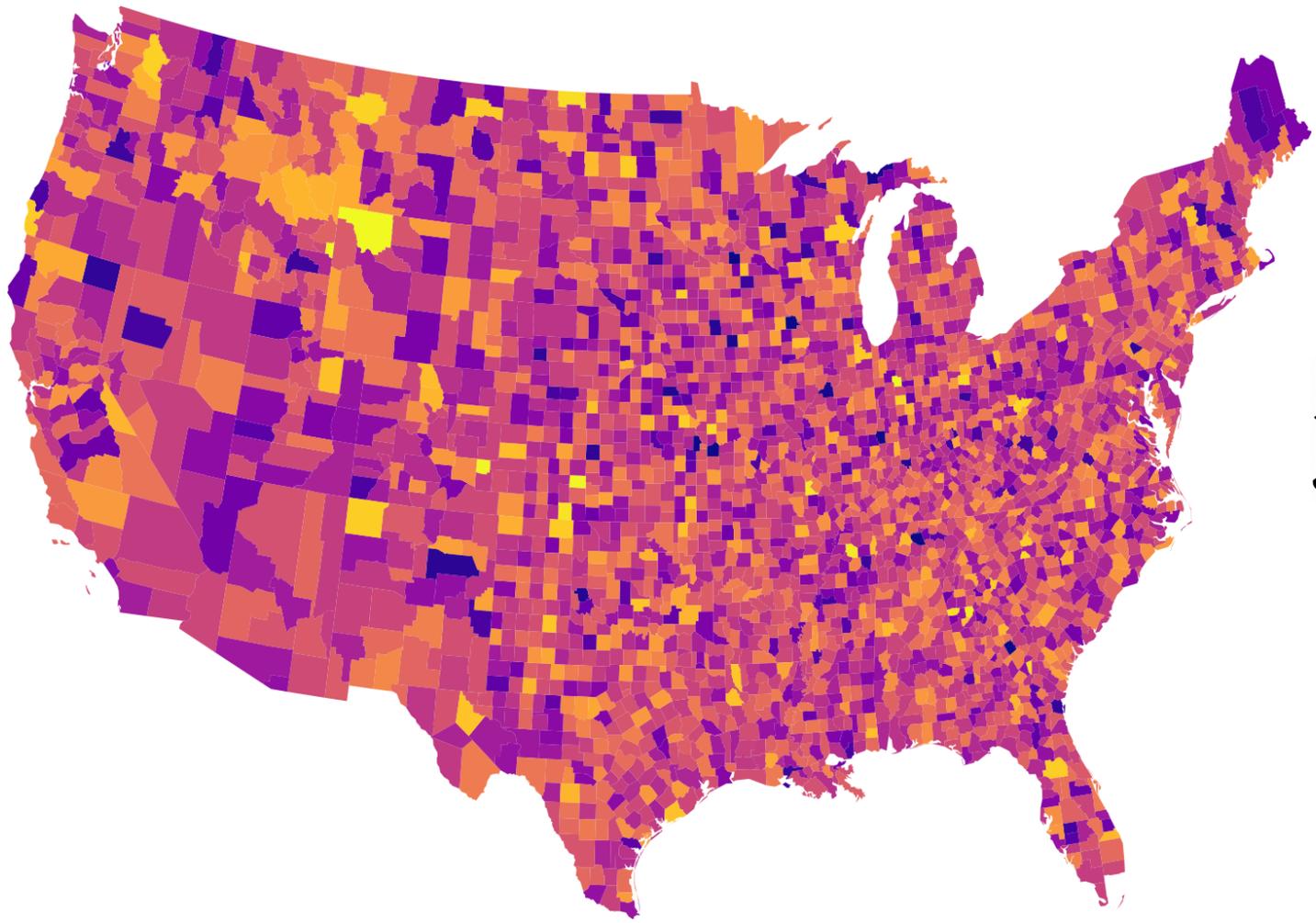
## **DEFINING A CLASSIC MODEL**

actually, we'll need a few

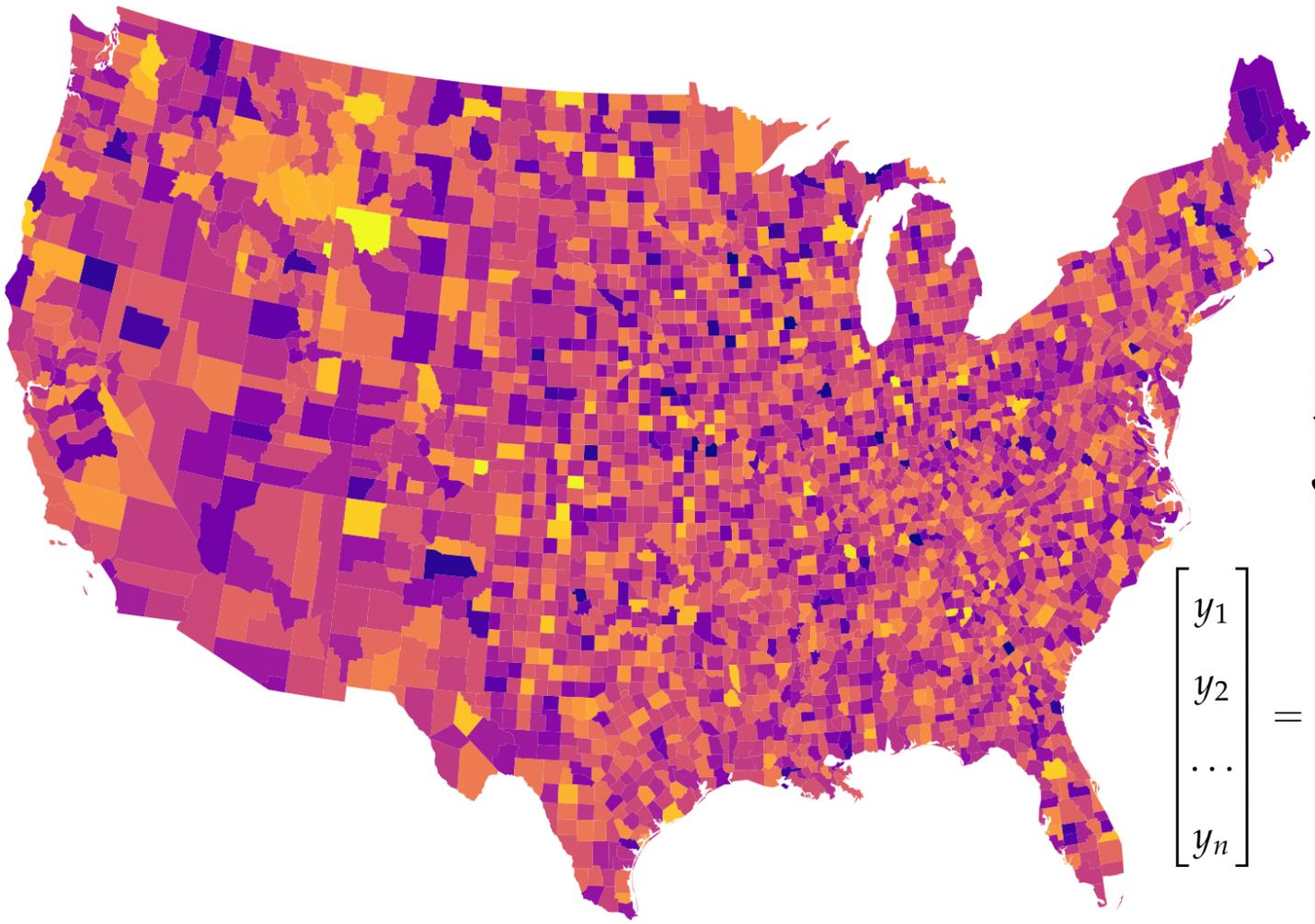
**IS PLACE JUST ABOUT GROUP?**

**DOES IT REALLY MATTER?**



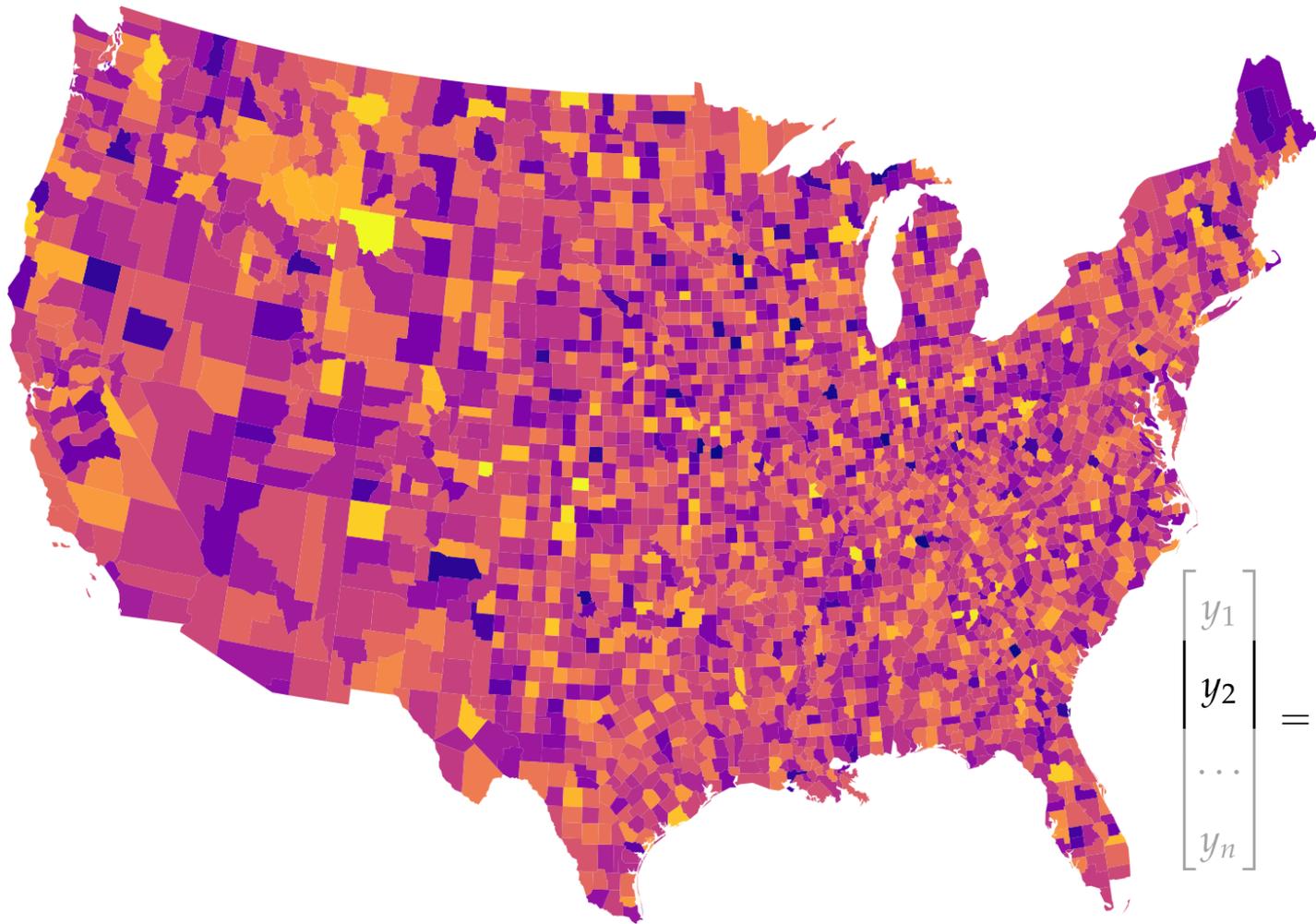


$$y = X\beta + \epsilon$$



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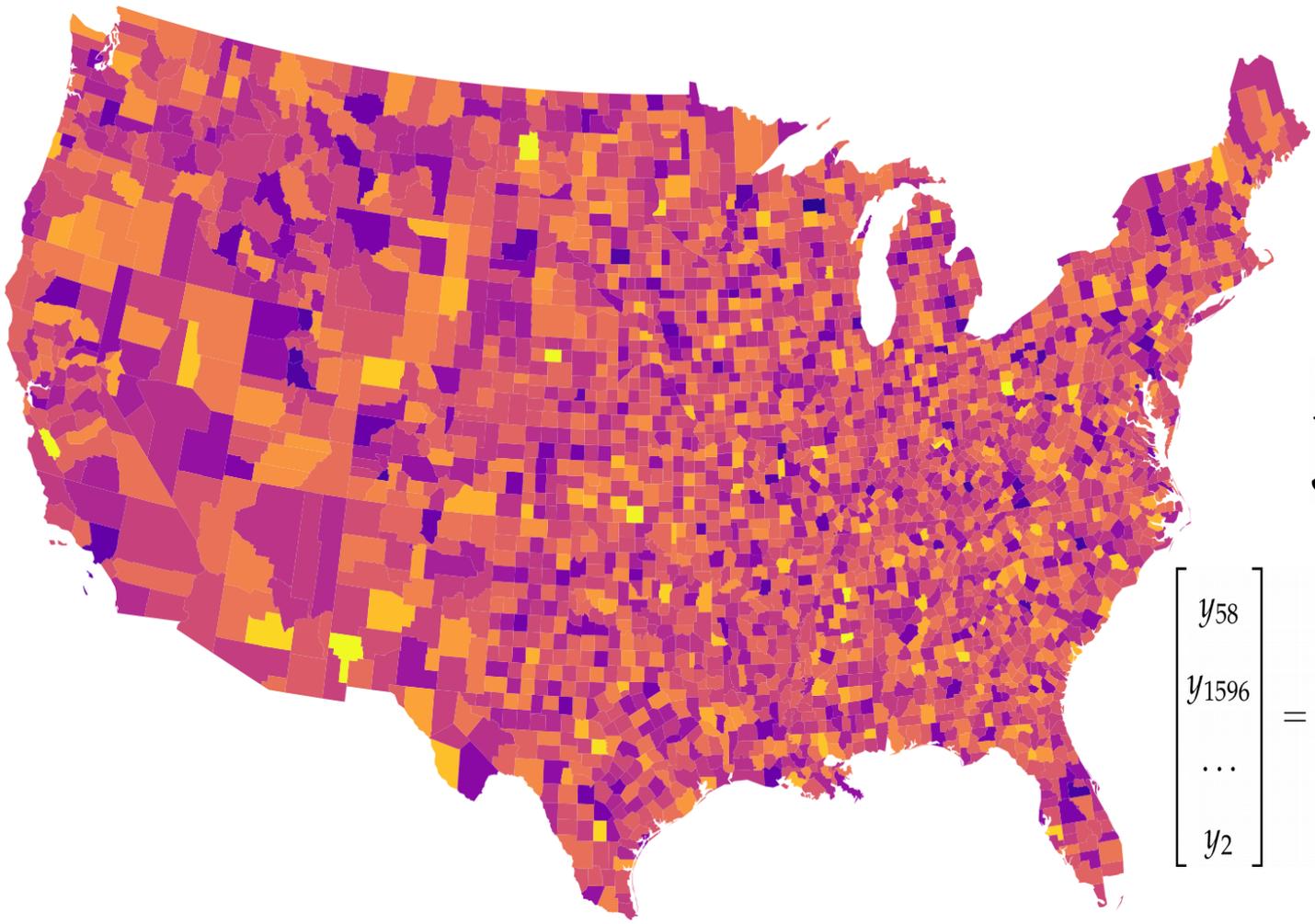
$$\begin{bmatrix} y_1 \\ y_2 \\ \dots \\ y_n \end{bmatrix} = \begin{bmatrix} x_{1,1} & x_{1,2} & \dots & x_{1,P} \\ x_{2,1} & x_{2,2} & \dots & x_{2,P} \\ \vdots & & \ddots & \vdots \\ x_{N,1} & x_{N,2} & \dots & x_{N,P} \end{bmatrix} \beta + \begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \vdots \\ \epsilon_N \end{bmatrix}$$



$$y = X\beta + \epsilon$$

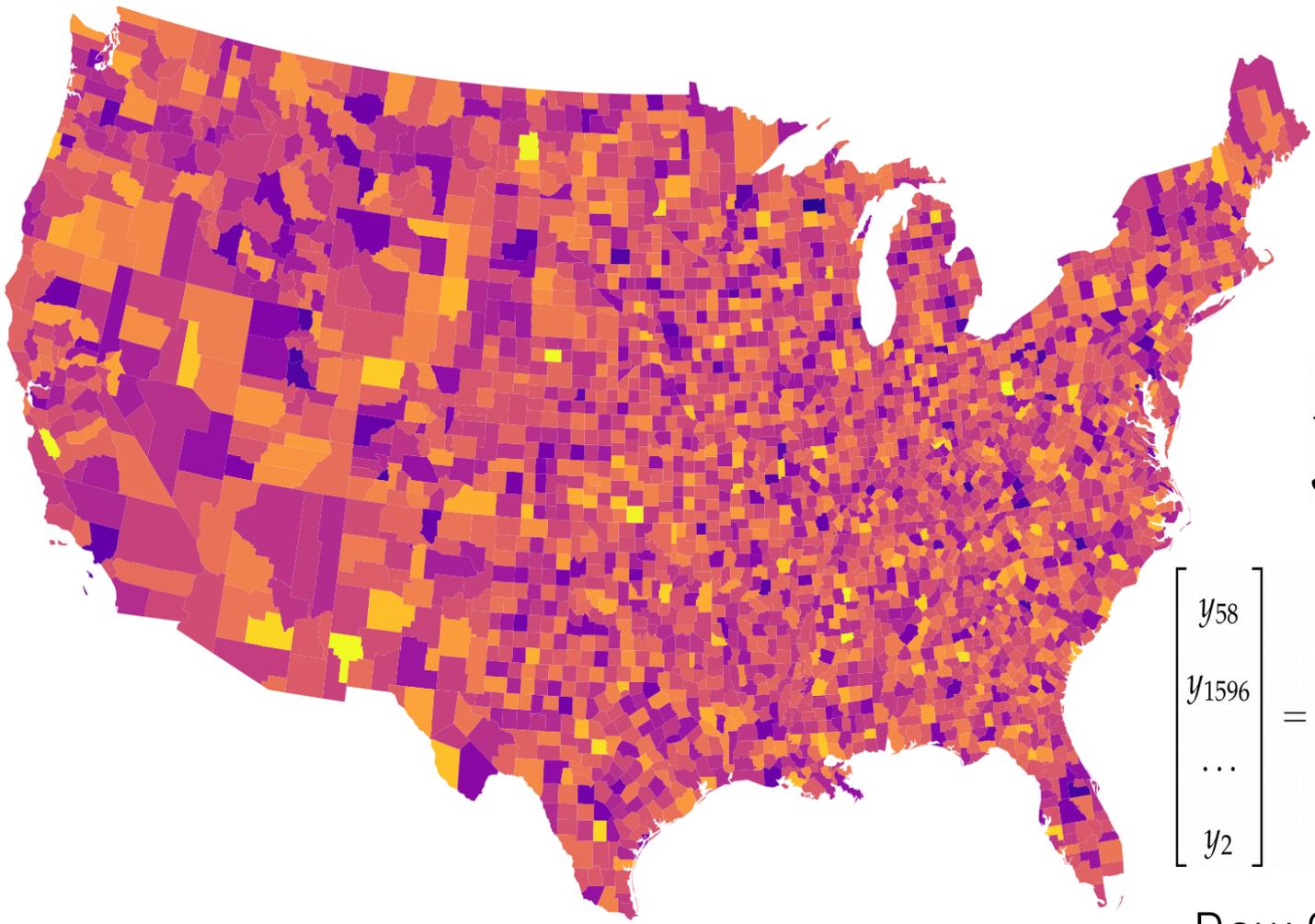
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Outcome at site 2 is a function of our measurements at site 2, plus a random error term



$$y = X\beta + \epsilon$$

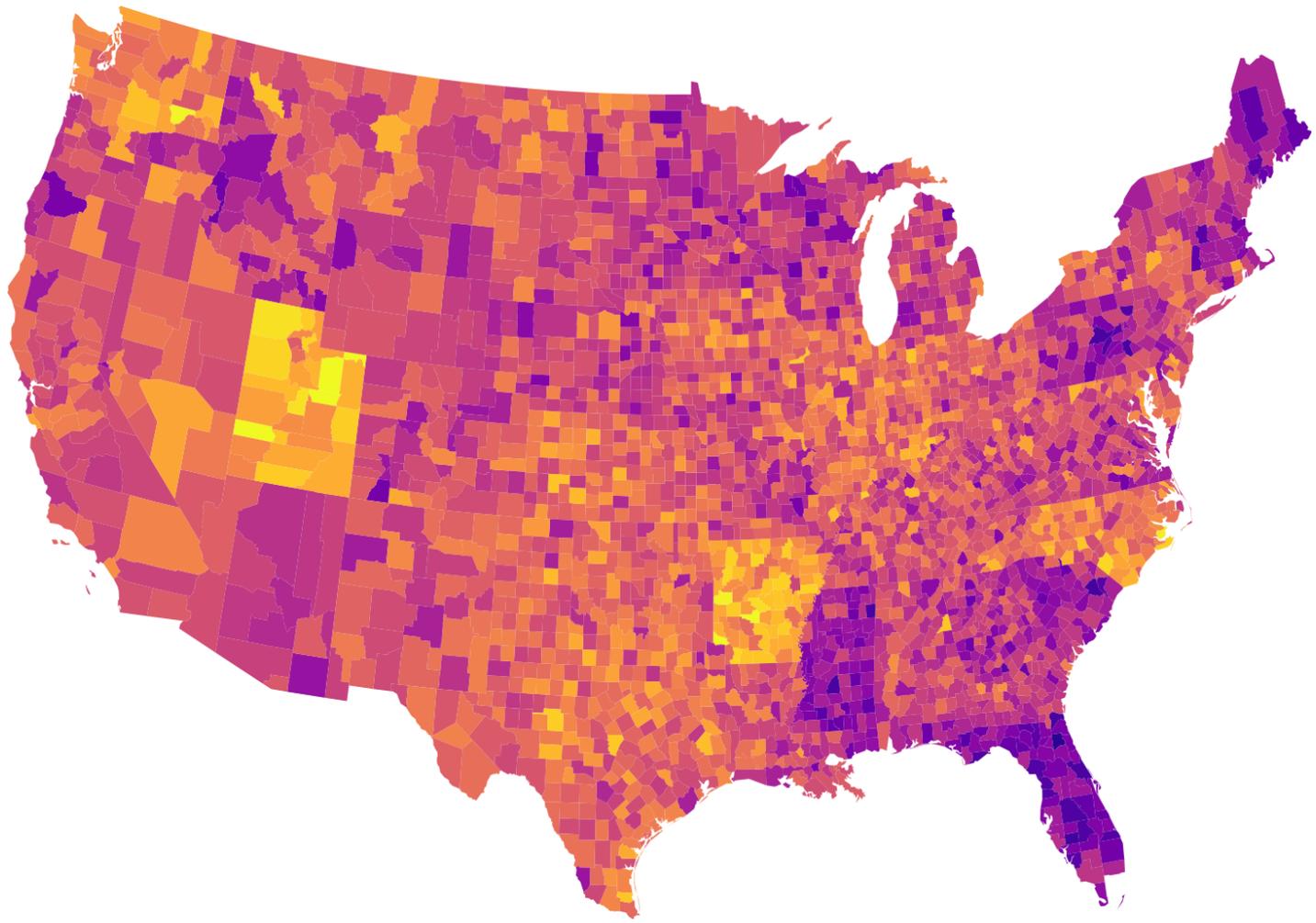
$$\begin{bmatrix} y_{58} \\ y_{1596} \\ \dots \\ y_2 \end{bmatrix} = \begin{bmatrix} x_{58,1} & x_{58,2} & \dots & x_{58,P} \\ x_{1596,1} & x_{1596,2} & \dots & x_{1596,P} \\ \vdots & & \ddots & \vdots \\ x_{2,1} & x_{2,2} & \dots & x_{2,P} \end{bmatrix} \beta + \begin{bmatrix} \epsilon_{58} \\ \epsilon_{1596} \\ \vdots \\ \epsilon_2 \end{bmatrix}$$

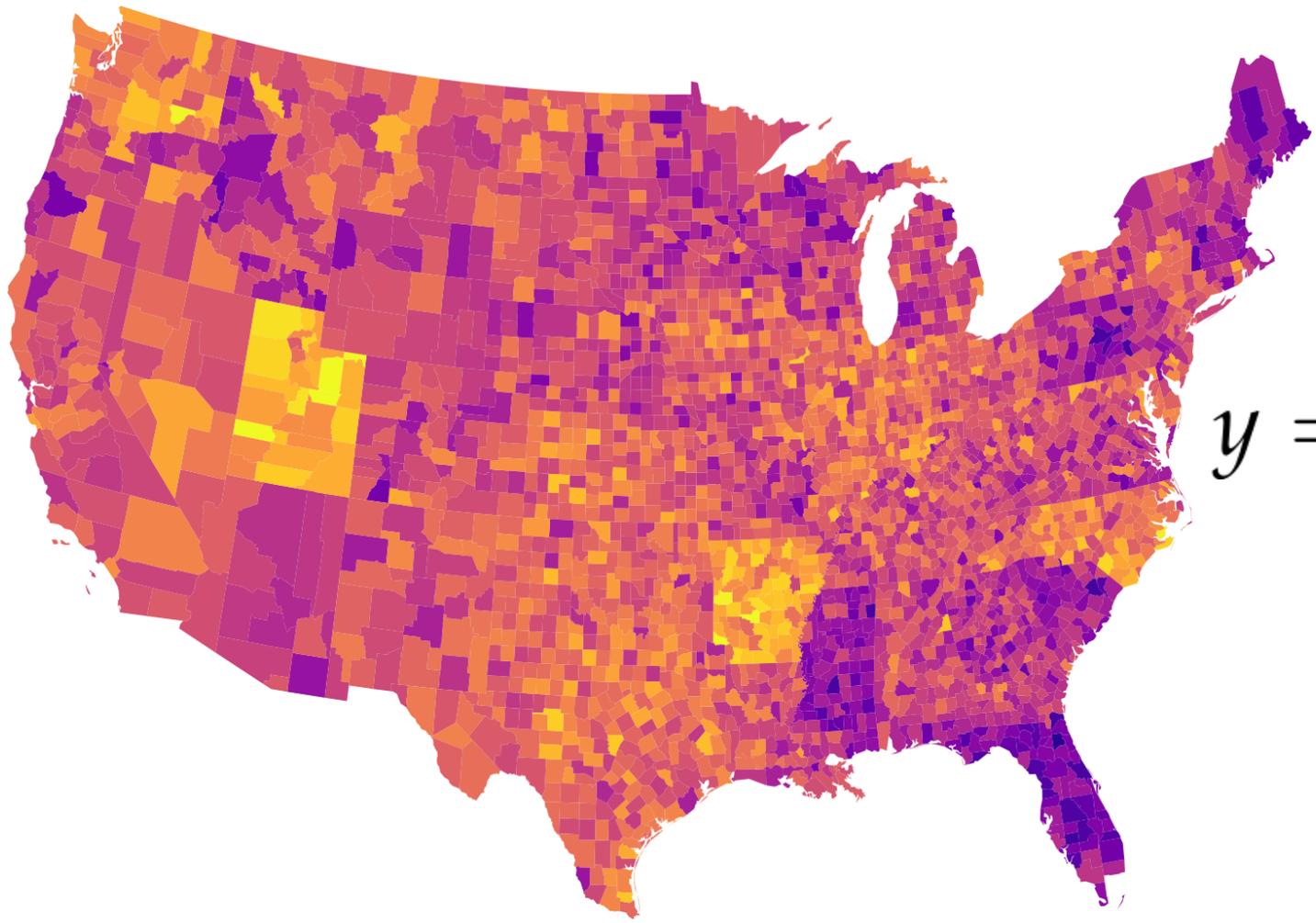


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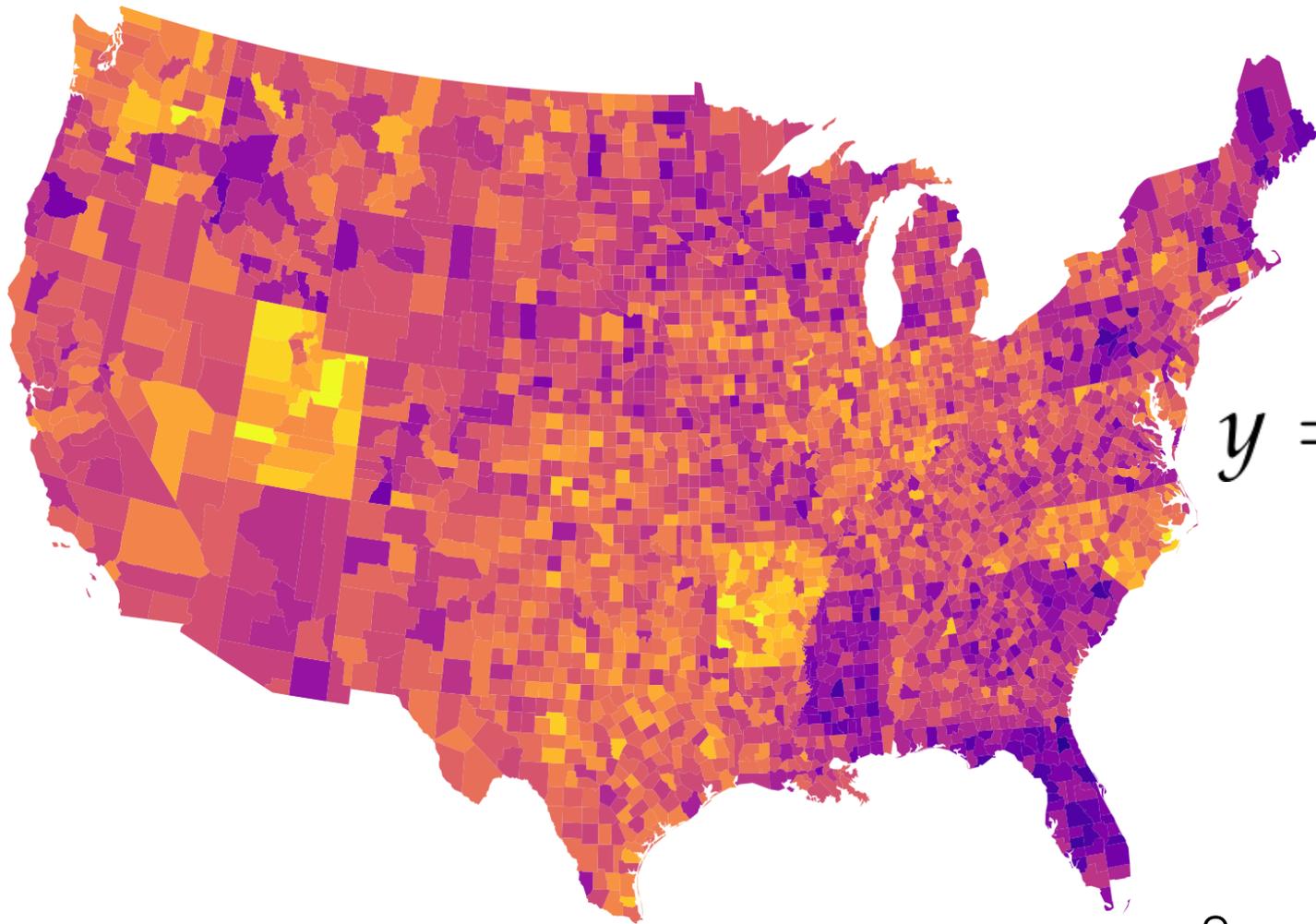
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Row 2 is no longer site 2!  
The geography has changed!  
The model can't "see" this!



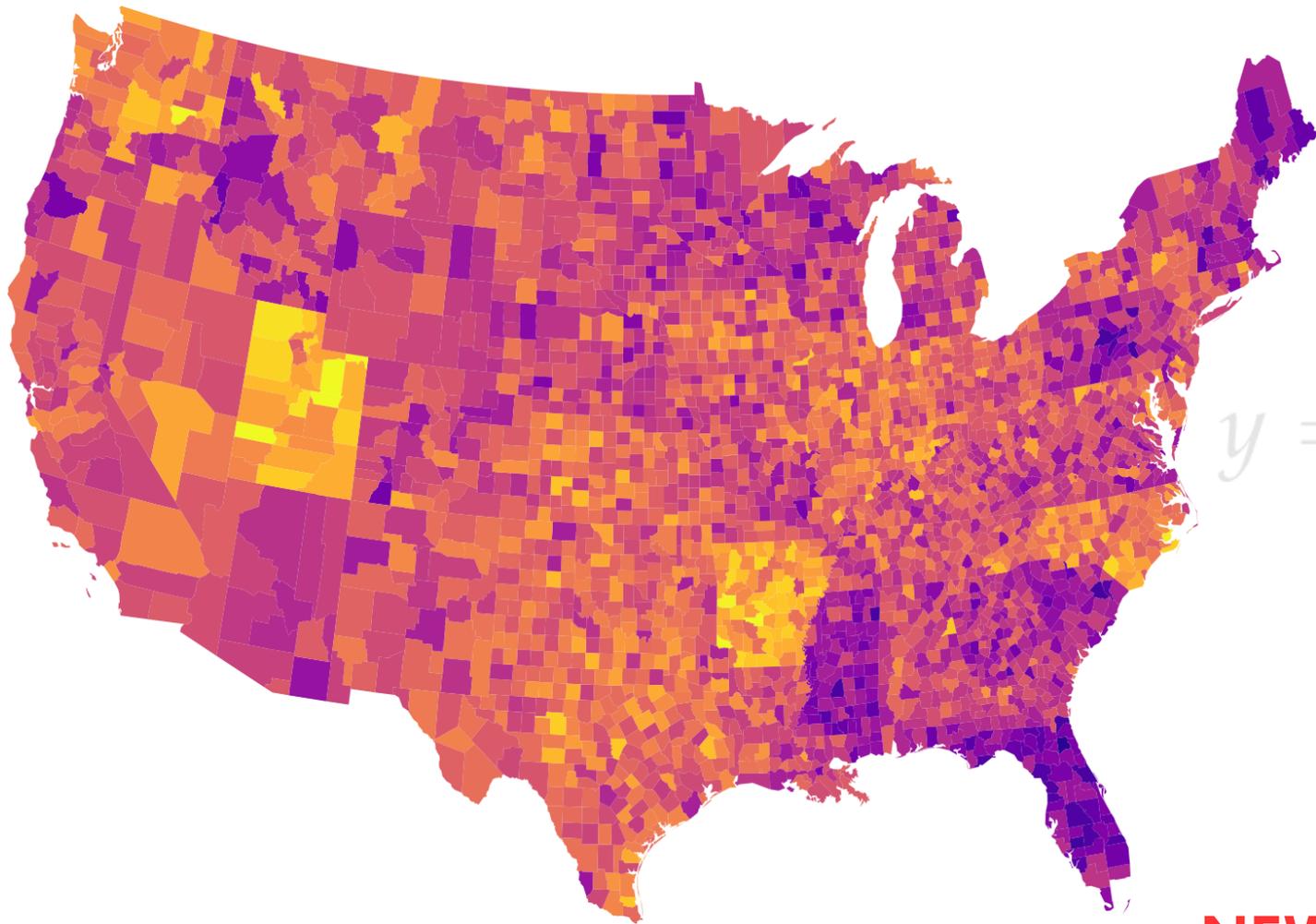


$$y = X\beta + \Delta u + \epsilon$$



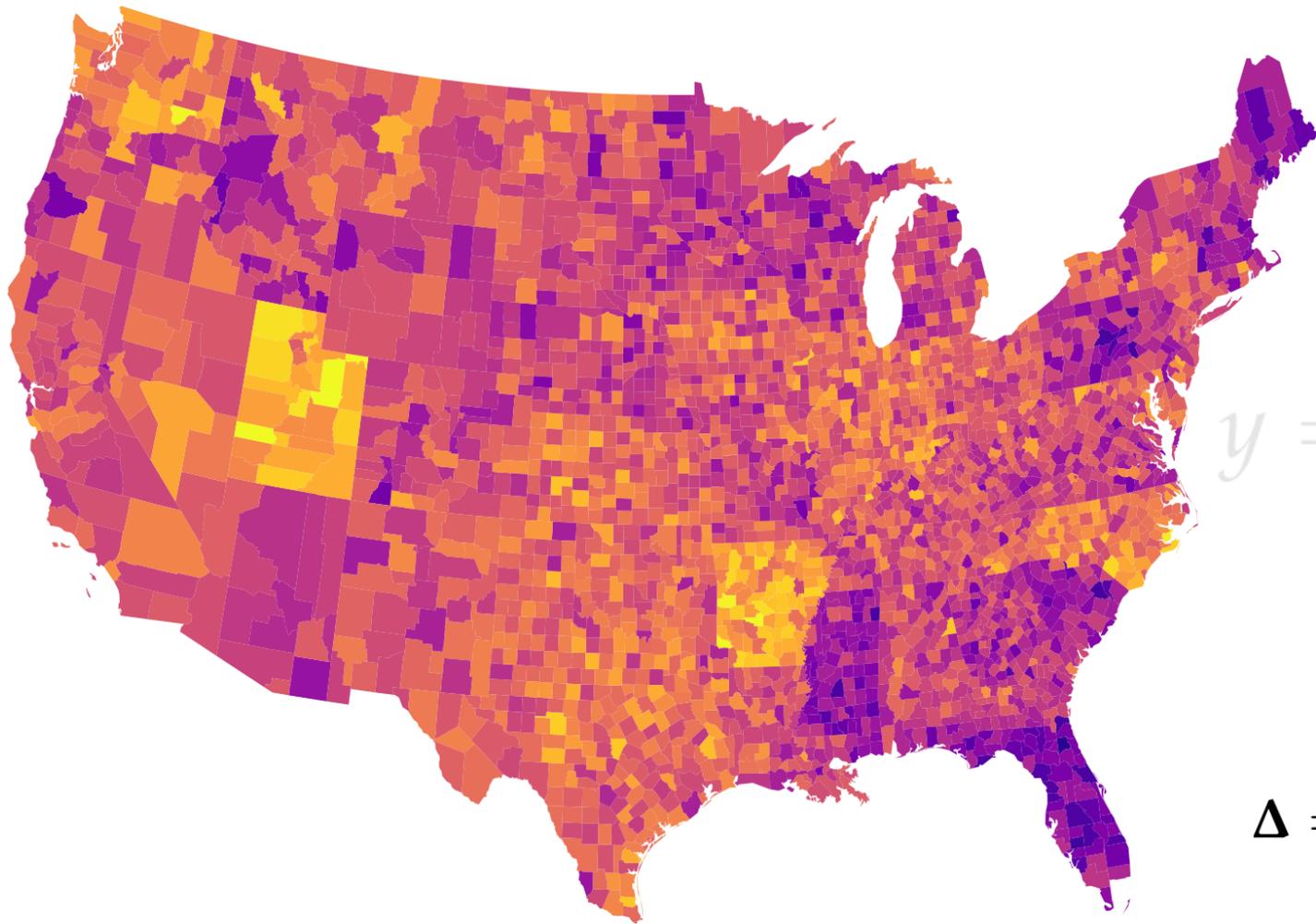
$$y = \mathbf{X}\beta + \Delta u + \epsilon$$

Same individual level factors  
we saw before



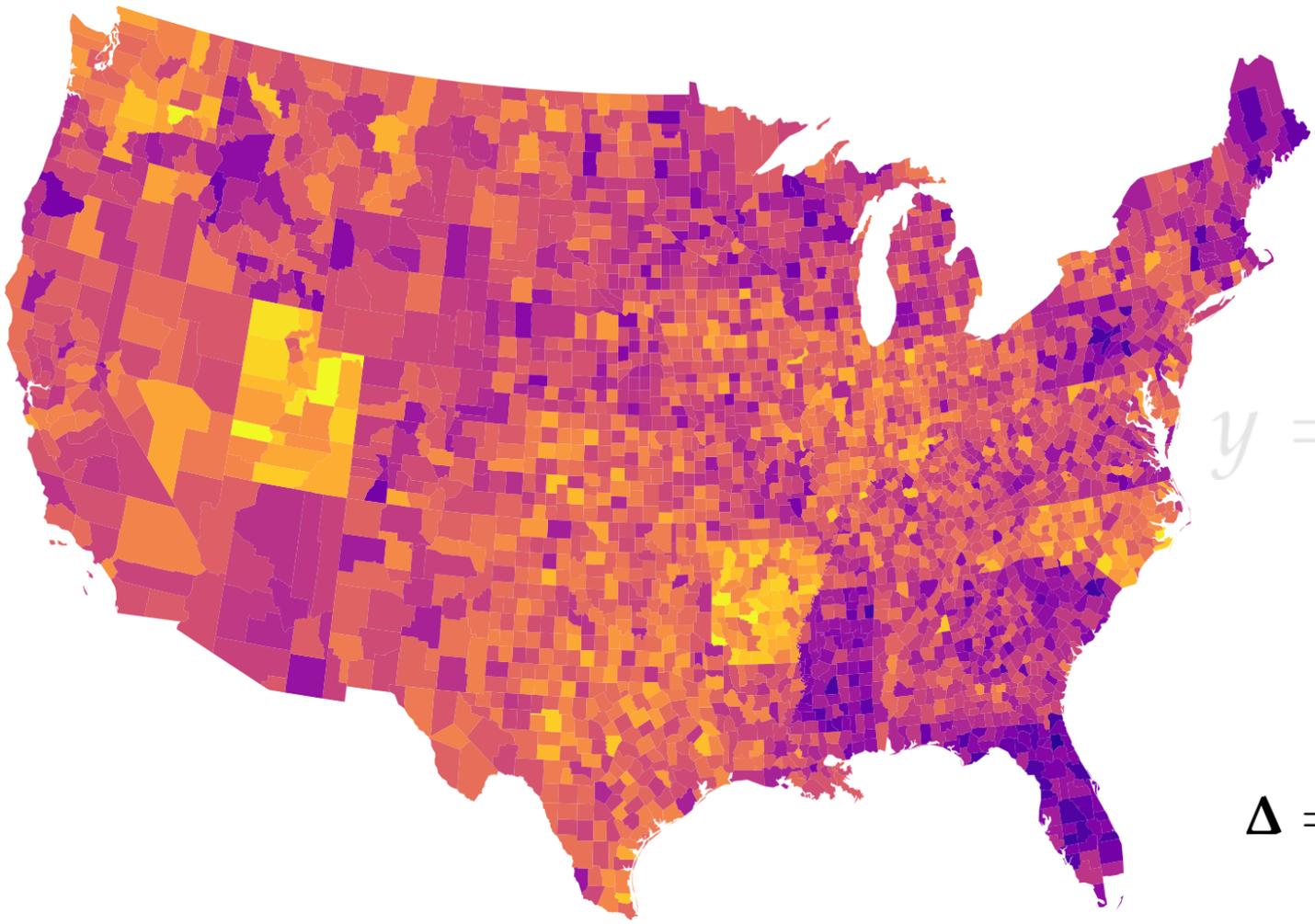
$$y = X\beta + \Delta u + \epsilon$$

**NEW** group-level factors  
that pertain to each “group”



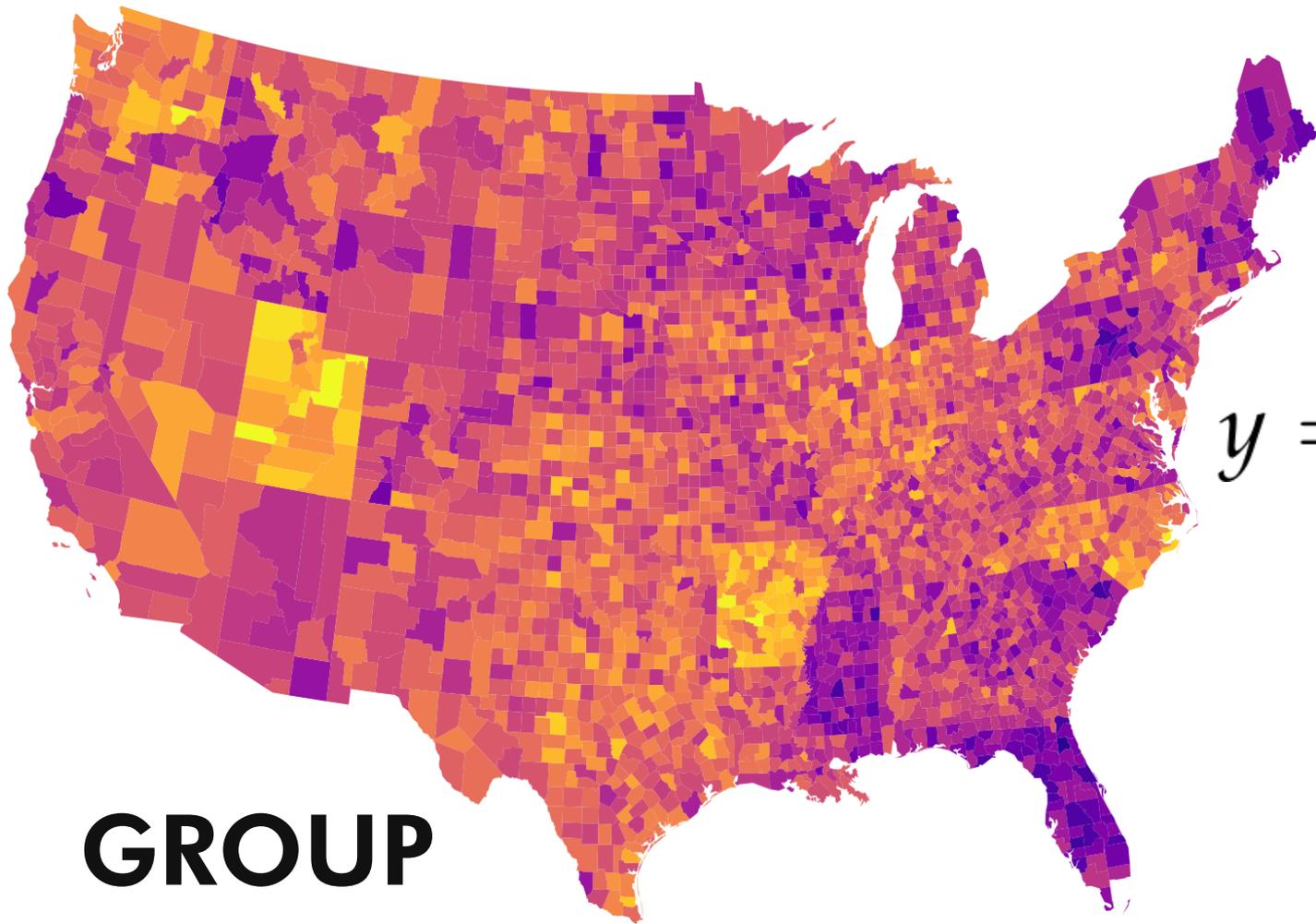
$$y = X\beta + \Delta u + \epsilon$$

$$\Delta = \begin{bmatrix} 1 & 0 & 0 & \dots & 0 \\ 1 & 0 & 0 & \dots & 0 \\ 0 & 1 & 0 & \dots & 0 \\ \vdots & & & \ddots & \vdots \end{bmatrix}$$



$$y = X\beta + \Delta u + \epsilon$$

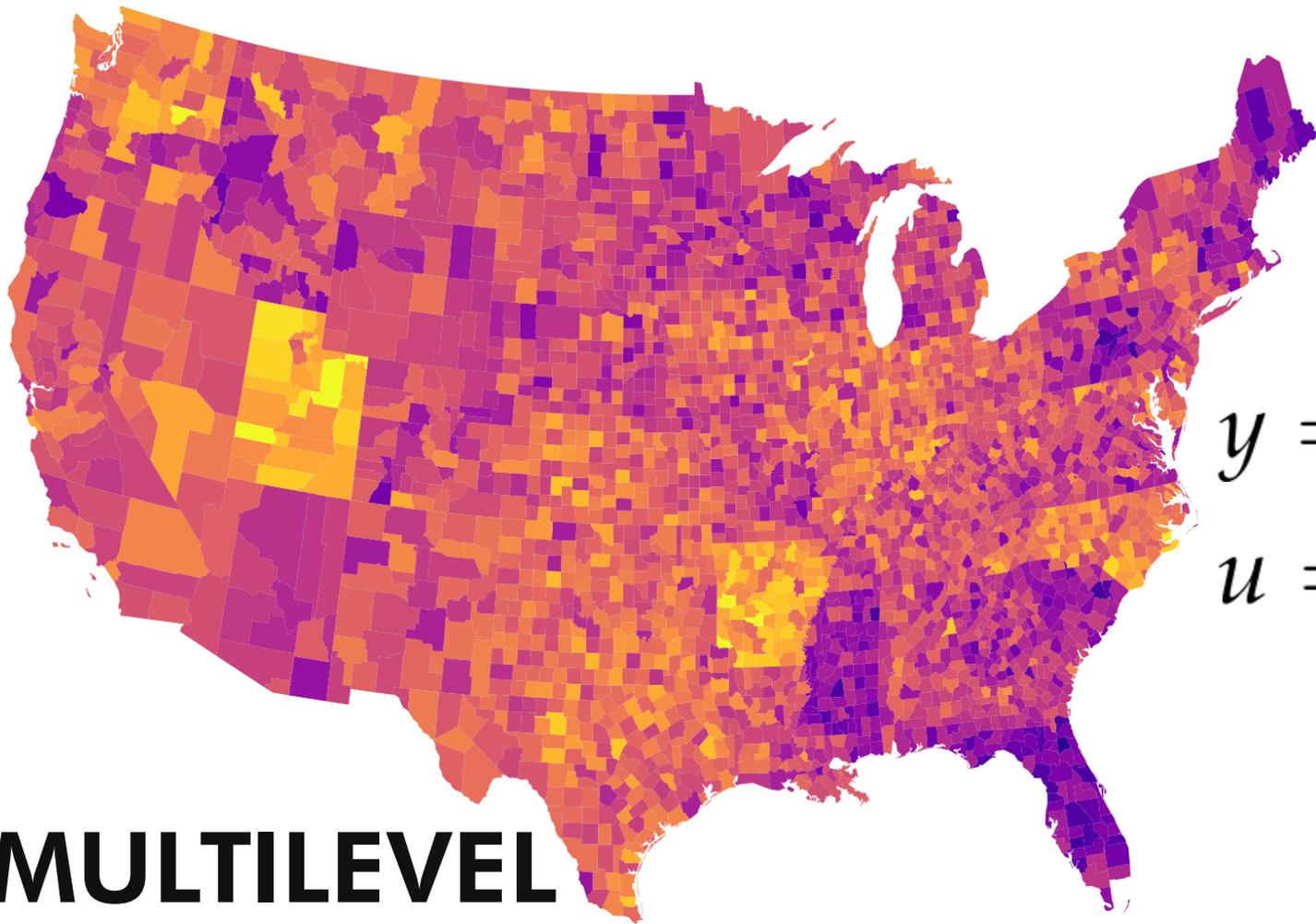
$$\Delta = \begin{bmatrix} 1 & 0 & 0 & \dots & 0 \\ 1 & 0 & 0 & \dots & 0 \\ 0 & 1 & 0 & \dots & 0 \\ \vdots & & & \ddots & \vdots \end{bmatrix} \begin{matrix} \text{AZ} \\ \text{TX} \\ \end{matrix}$$



$$y = X\beta + \Delta u + \epsilon$$

# GROUP FIXED EFFECTS

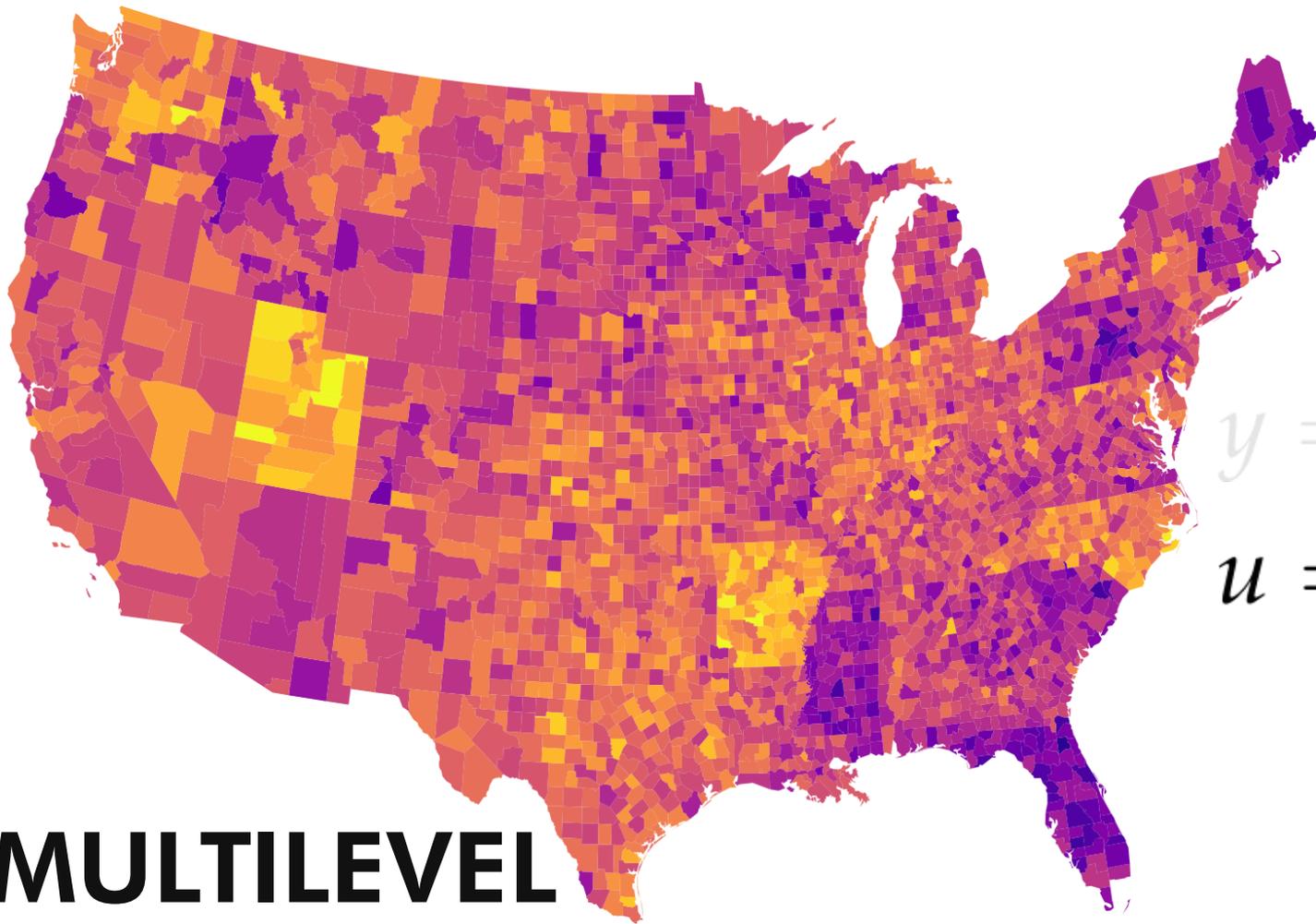
Give groups separate intercepts.



$$y = \mathbf{X}\beta + \Delta u + \epsilon$$
$$u = a + \mathbf{Z}\gamma + \zeta$$

# MULTILEVEL VARYING INTERCEPTS

Give groups separate intercepts.  
Have a theory as to why/how they're different.



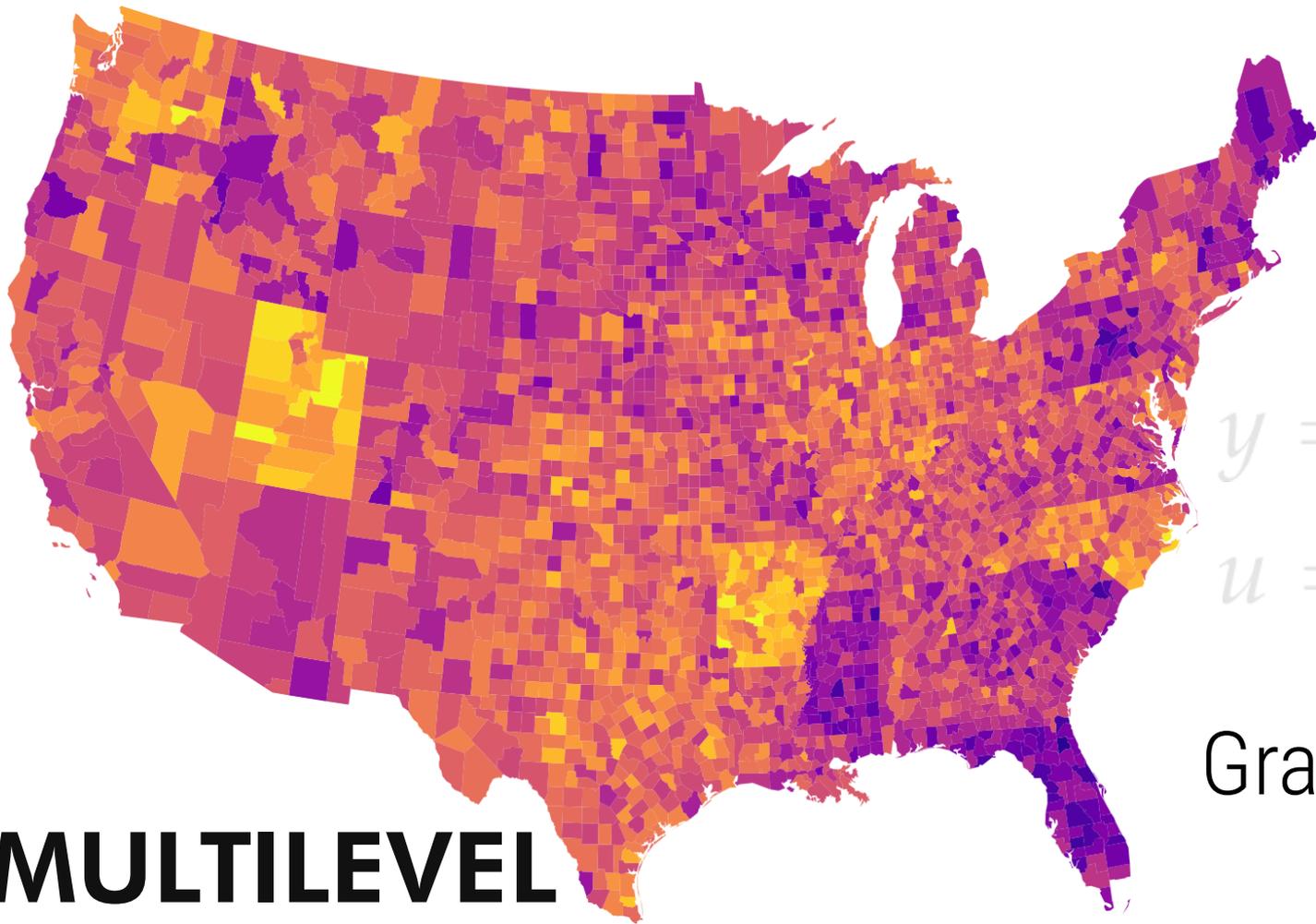
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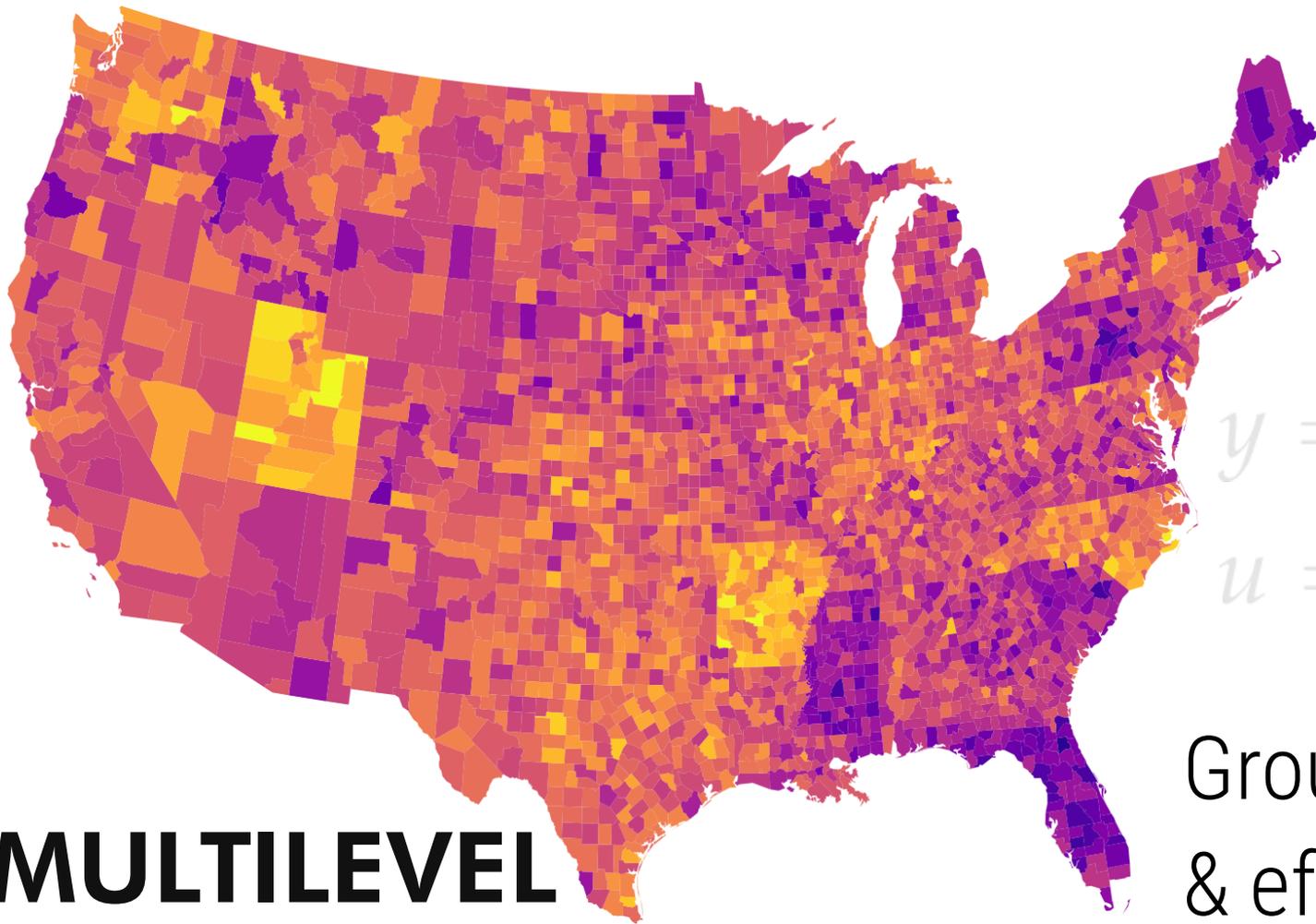
$$u = a + Z\gamma + \zeta$$

Grand mean

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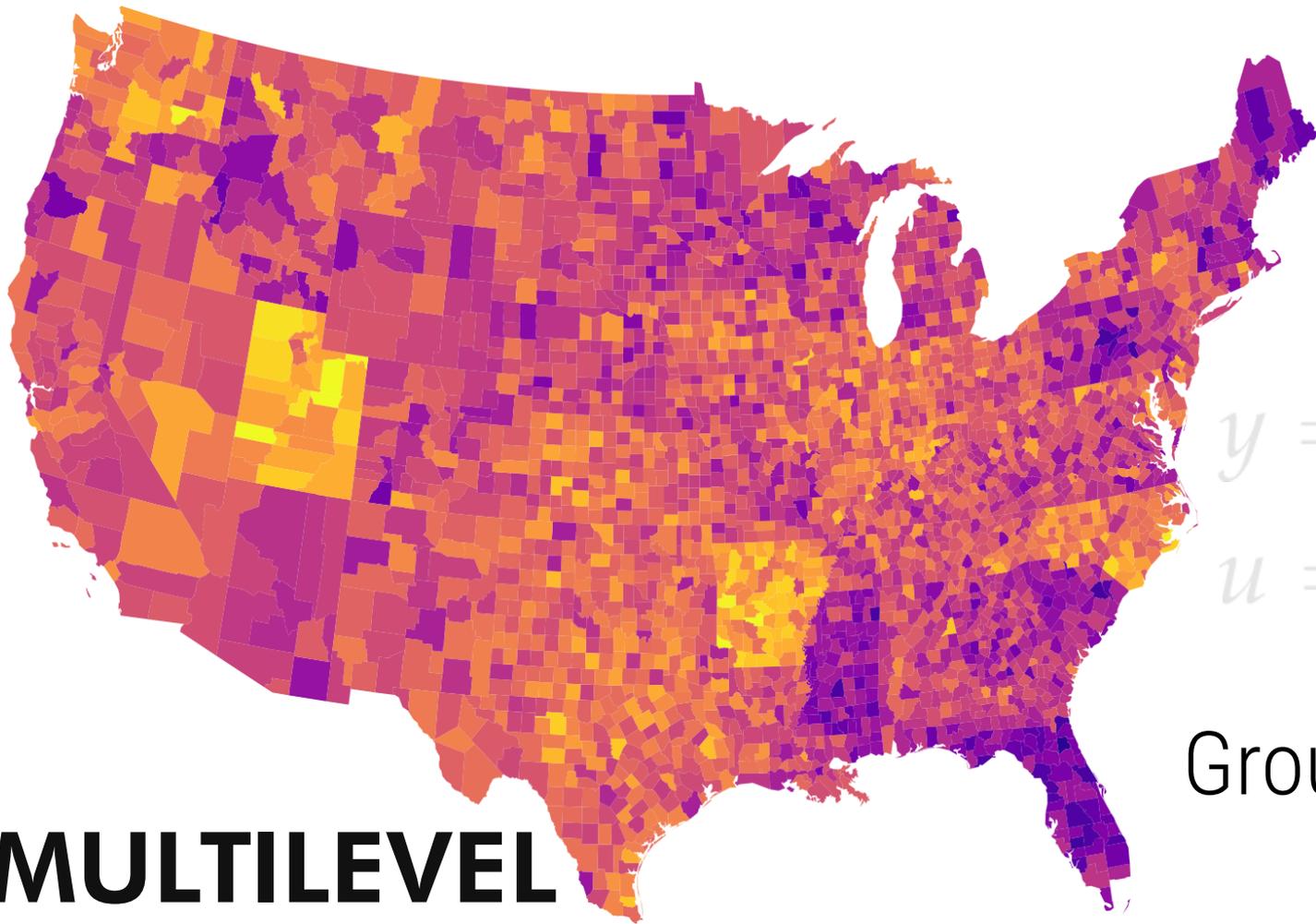


# MULTILEVEL VARYING INTERCEPTS

$$y = X\beta + \Delta u + \epsilon$$
$$u = a + \mathbf{Z}\gamma + \zeta$$

Group-level data ( $\mathbf{Z}$ )  
& effects ( $\gamma$ )

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$$y = X\beta + \Delta u + \epsilon$$

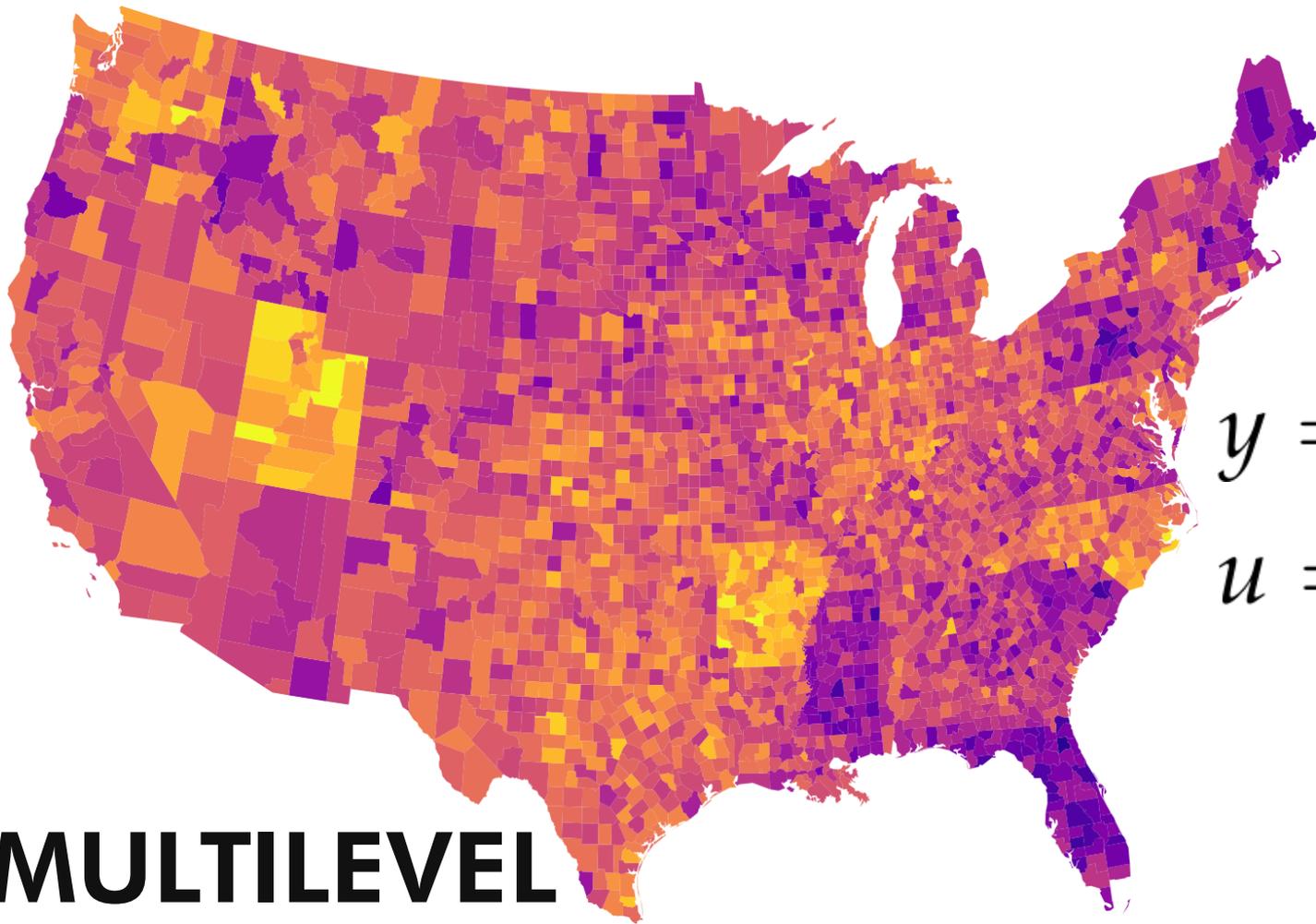
$$u = a + Z\gamma + \zeta$$

Group-level error

# MULTILEVEL VARYING INTERCEPTS

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$$y = \Delta u + \epsilon$$
$$u = a + \zeta$$

# MULTILEVEL VARYING INTERCEPTS

Even without  $\mathbf{Z}, \mathbf{X}$ , these give more precise but biased estimates (Gelman & Hill, 2006)

$$\hat{u}_{j,ols} = \frac{\sum_{i \in j} y_i}{n_j}$$

**FIXED  
EFFECT  
ESTIMATES**

The group intercept is the average of observations within the group.

(Gelman & Hill, 2006)

$$\hat{u}_{j,mlm} = \frac{\sigma_u^2}{\sigma_u^2 + \frac{\sigma_e^2}{n_j}} \hat{u}_{j,ols}$$

The group intercept is the average of observations within the group,  
**biased towards zero!**

(Gelman & Hill, 2006)

**MULTILEVEL  
INTERCEPT  
ESTIMATES**

$$\hat{u}_{j,mlm} = \frac{\sigma_u^2}{\sigma_u^2 + \frac{\sigma_e^2}{n_j}} \hat{u}_{j,ols}$$

$n_j$  : Group size

$\sigma_e^2$  : w/in group variation

$\sigma_u^2$  : between group variation

(Gelman & Hill, 2006)

**SHRINKAGE  
FACTOR**

$$\hat{u}_{j,mlm} = \frac{\sigma_u^2}{\sigma_u^2 + \frac{\sigma_e^2}{n_j}} \hat{u}_{j,ols}$$

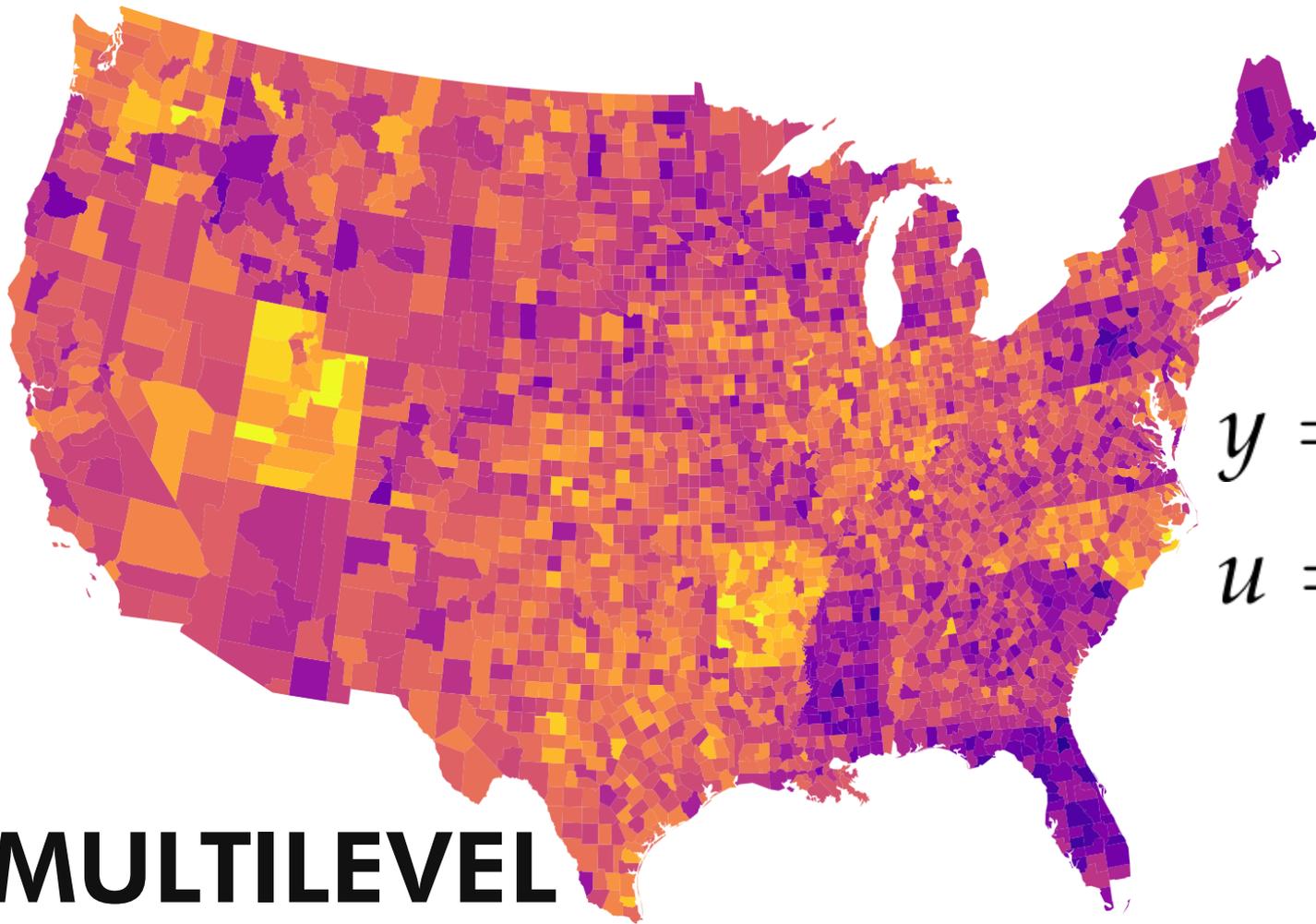
Small groups shrink more than big groups

Problems with noisy groups shrink more

Problems with similar groups shrink more

(Gelman & Hill, 2006)

## SHRINKAGE FACTOR



$$y = \Delta u + \epsilon$$
$$u = a + \zeta$$

# MULTILEVEL VARYING INTERCEPTS

Observations within my group  
are likely more similar to me than others.

# THE GEOGRAPHIC DICHOTOMY

space & place are core to geography

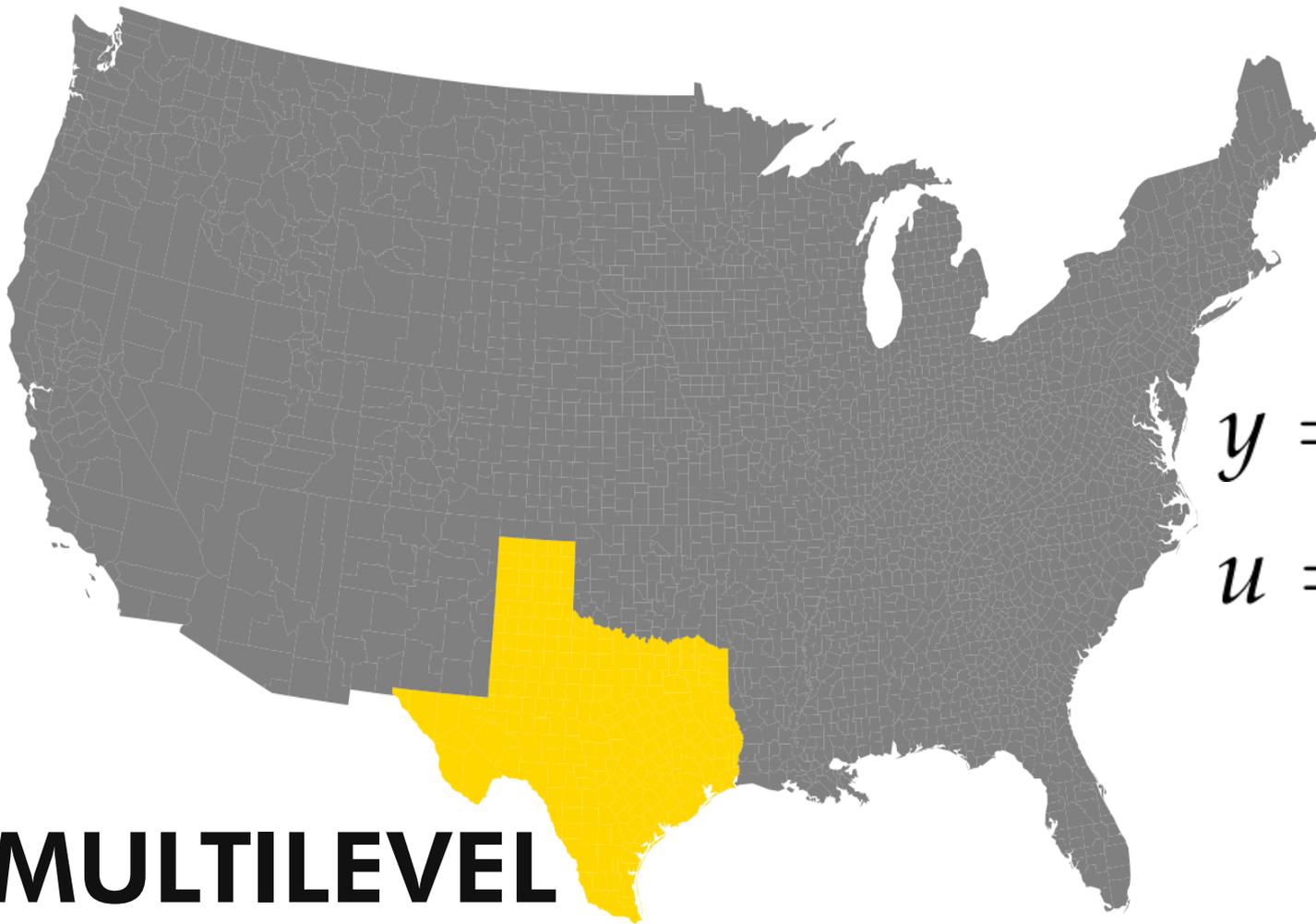
## DEFINING A CLASSIC MODEL

multilevel models use place to estimate better

## **IS PLACE JUST ABOUT GROUP?**

accidentally geographic models

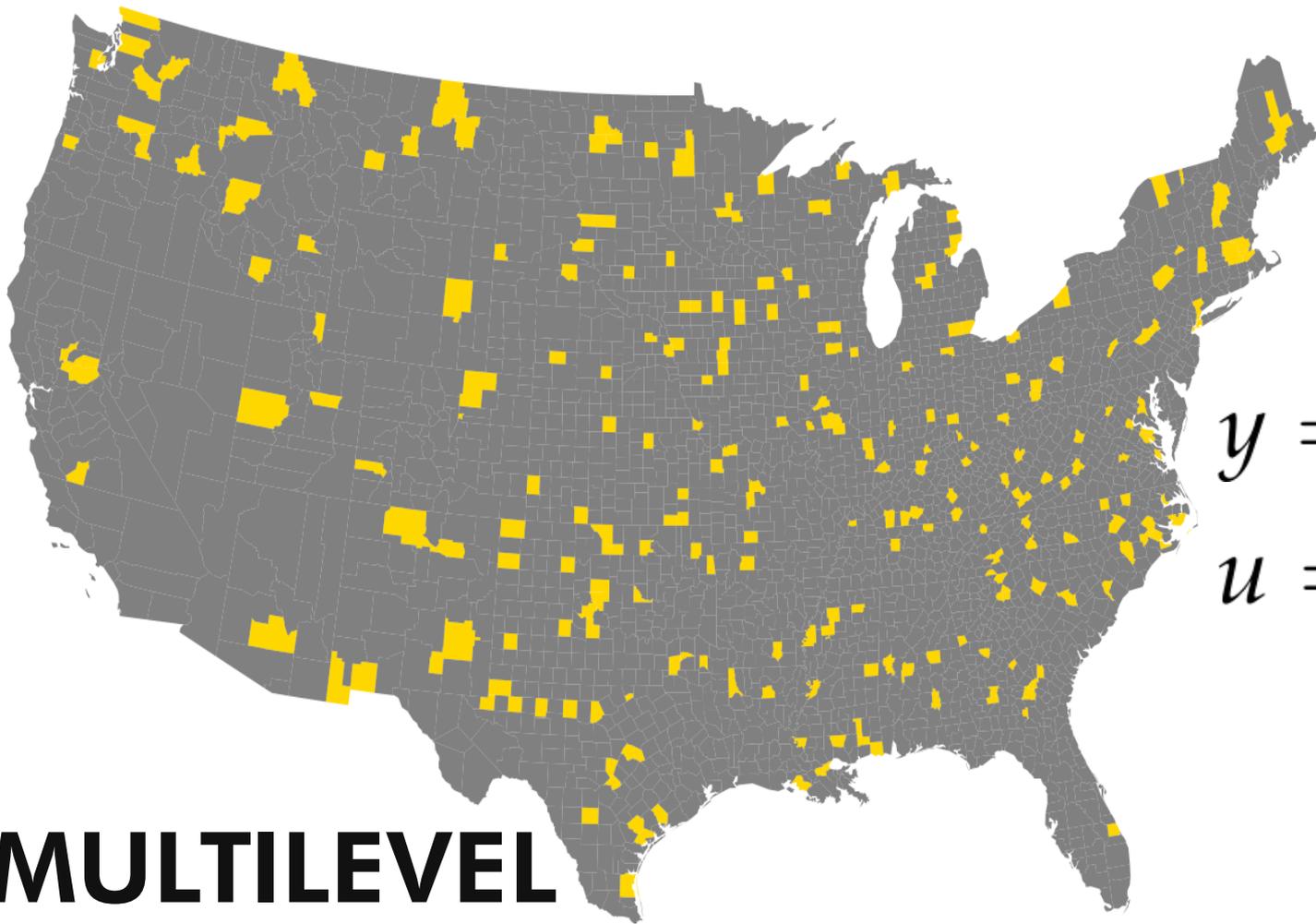
## DOES IT REALLY MATTER?



$$y = \Delta u + \epsilon$$
$$u = a + \zeta$$

# MULTILEVEL VARYING INTERCEPTS

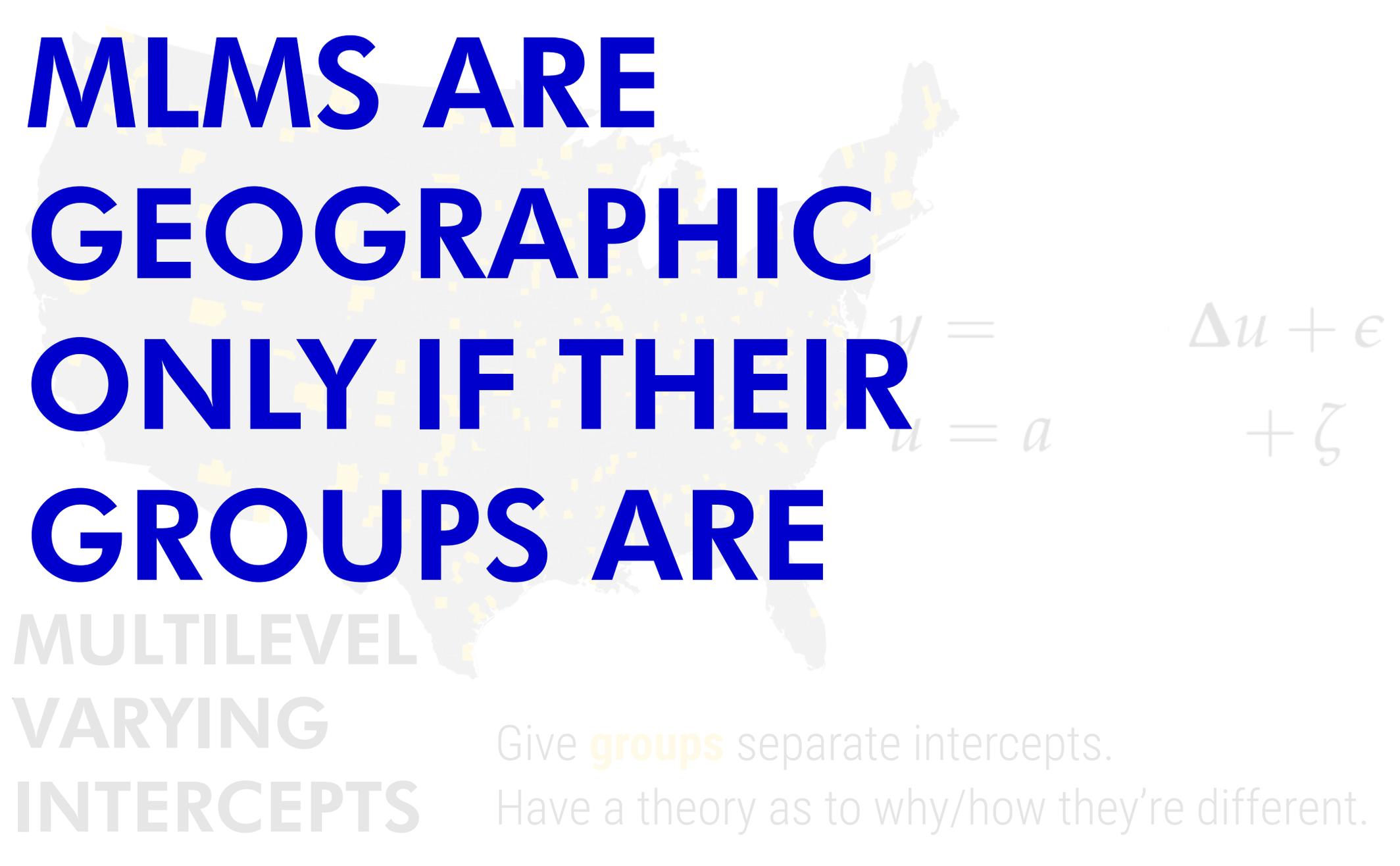
Give **groups** separate intercepts.  
Have a theory as to why/how they're different.



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# MLMS ARE GEOGRAPHIC ONLY IF THEIR GROUPS ARE

MULTILEVEL  
VARYING  
INTERCEPTS

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**MLMS ARE  
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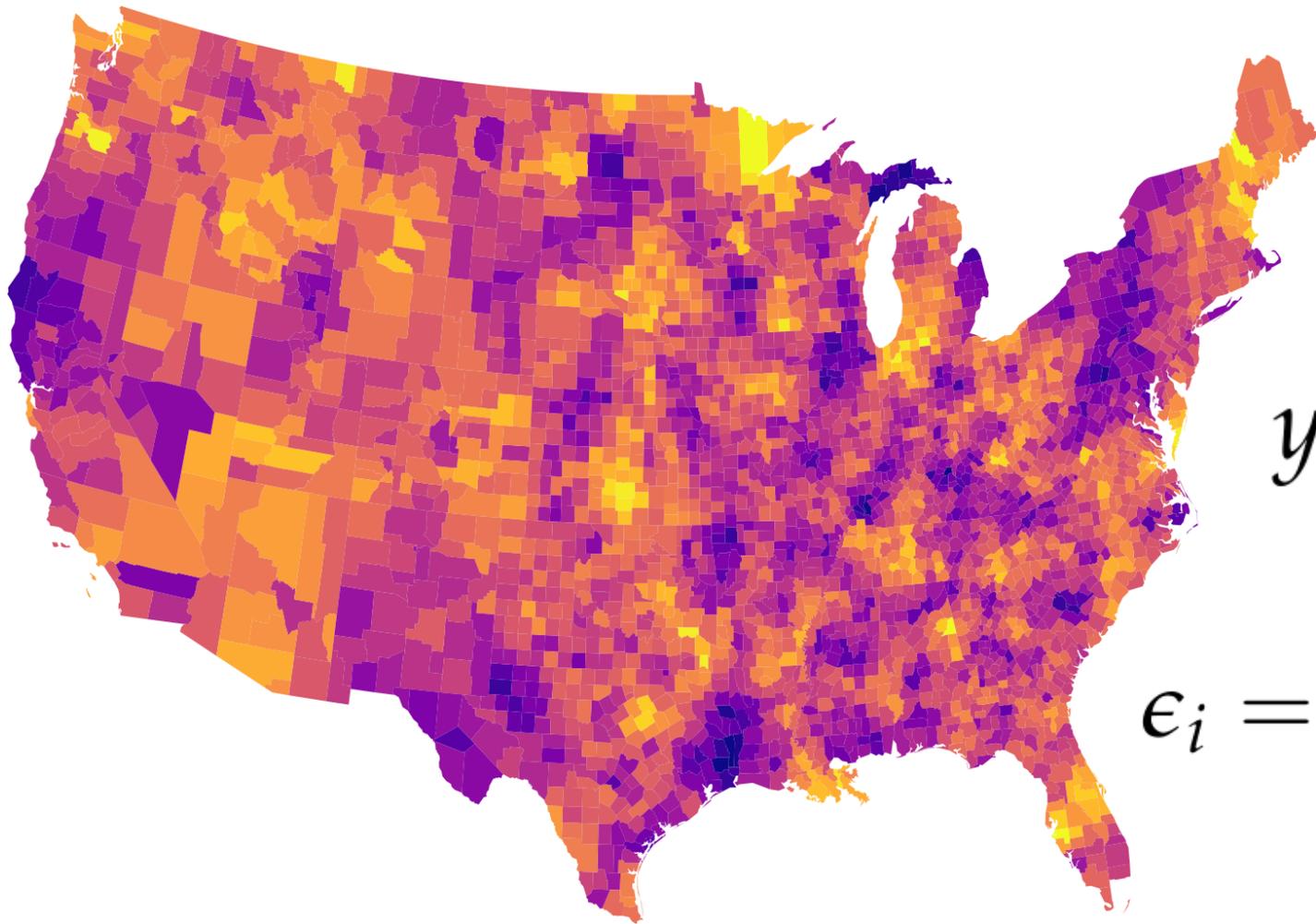
**(the model itself doesn't care!)**

Give **groups** separate intercepts.

Have a theory as to why/how they're different.

$$y = \Delta u + \epsilon$$
$$u = a + \zeta$$

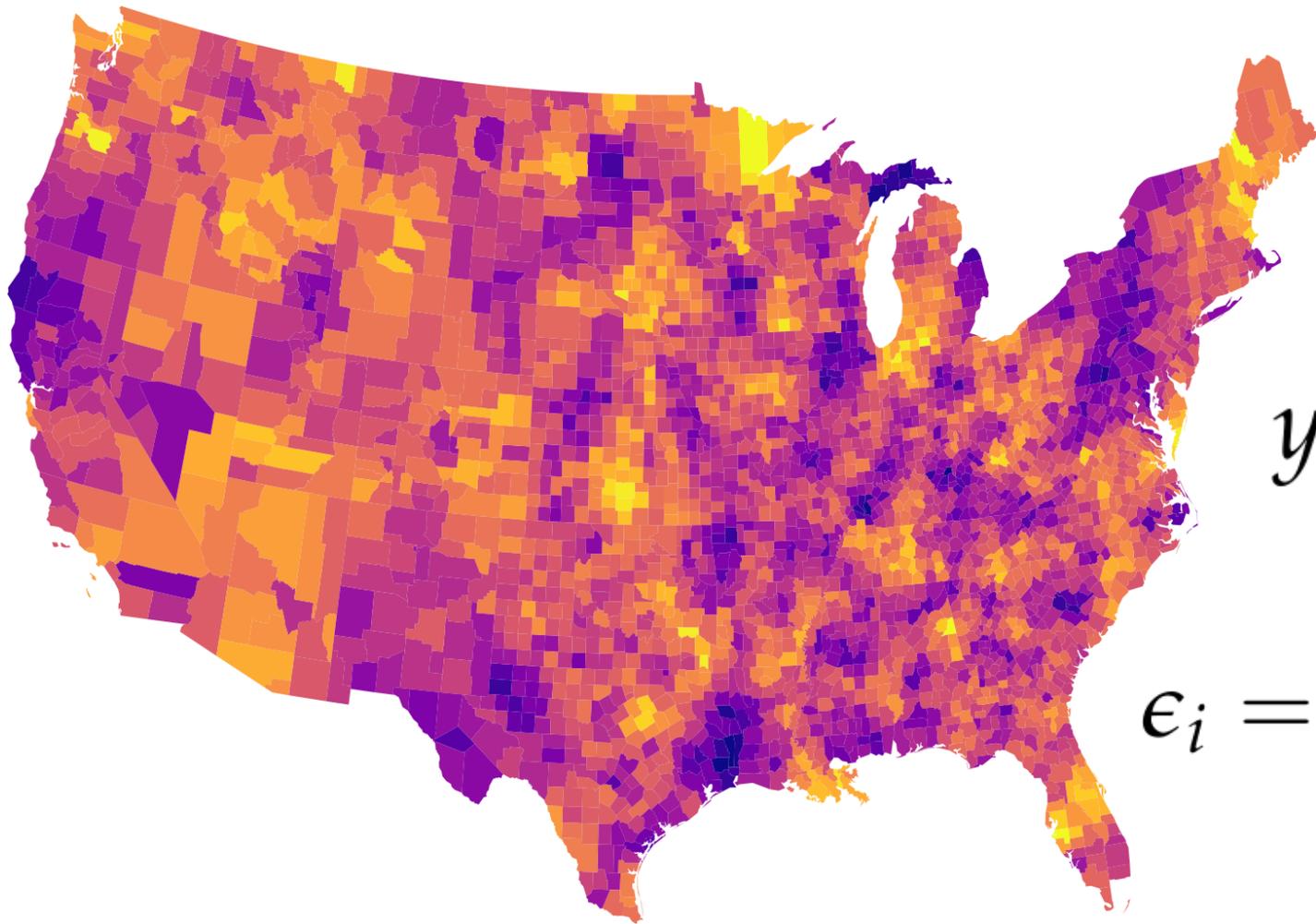
MULTILEVEL  
VARYING  
INTERCEPTS



$$y_i = u_{j[i]} + \epsilon_i$$

$$\epsilon_i = \lambda \sum_i^N w_{ij} \epsilon_j + e_i$$

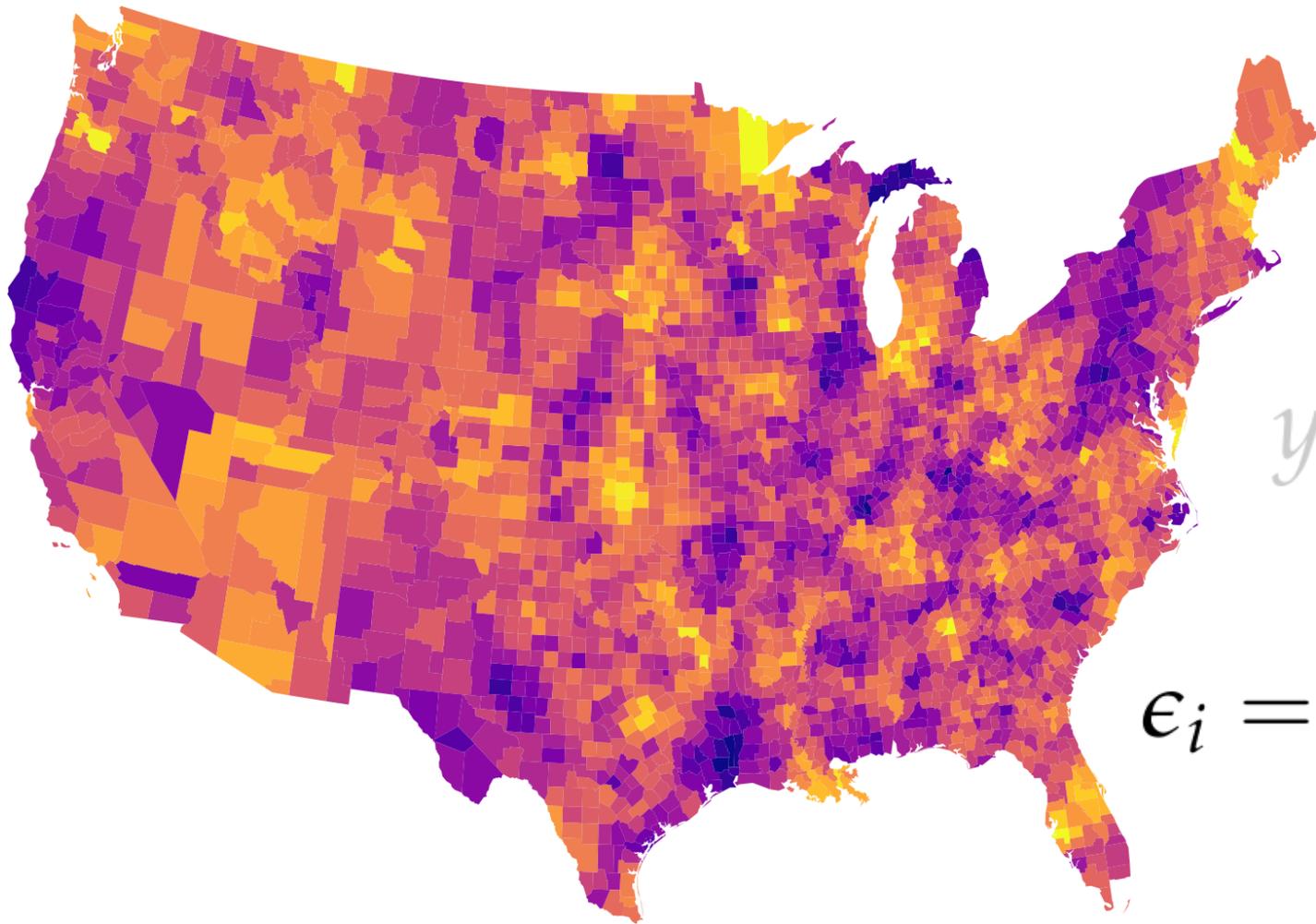
# **SIMULTANEOUS AUTOREGRESSIVE**



$$y_i = u_{j[i]} + \epsilon_i$$

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**SIMULTANEOUS  
AUTOREGRESSIVE**

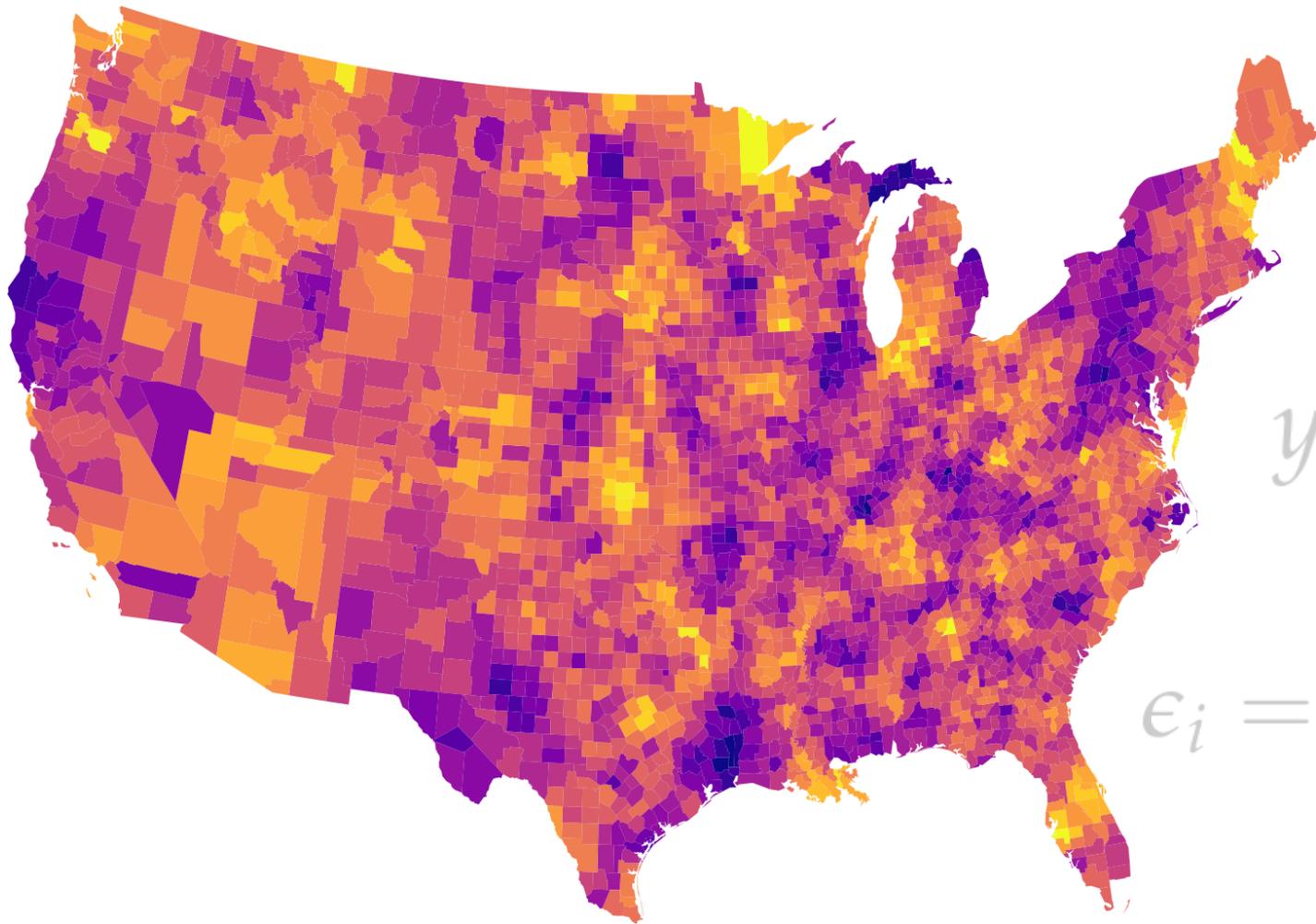


$$y_i = u_j[i] + \epsilon_i$$

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# SAR MODELS

Error at site  $i$  is an average of surrounding errors, plus intrinsic error at site  $i$

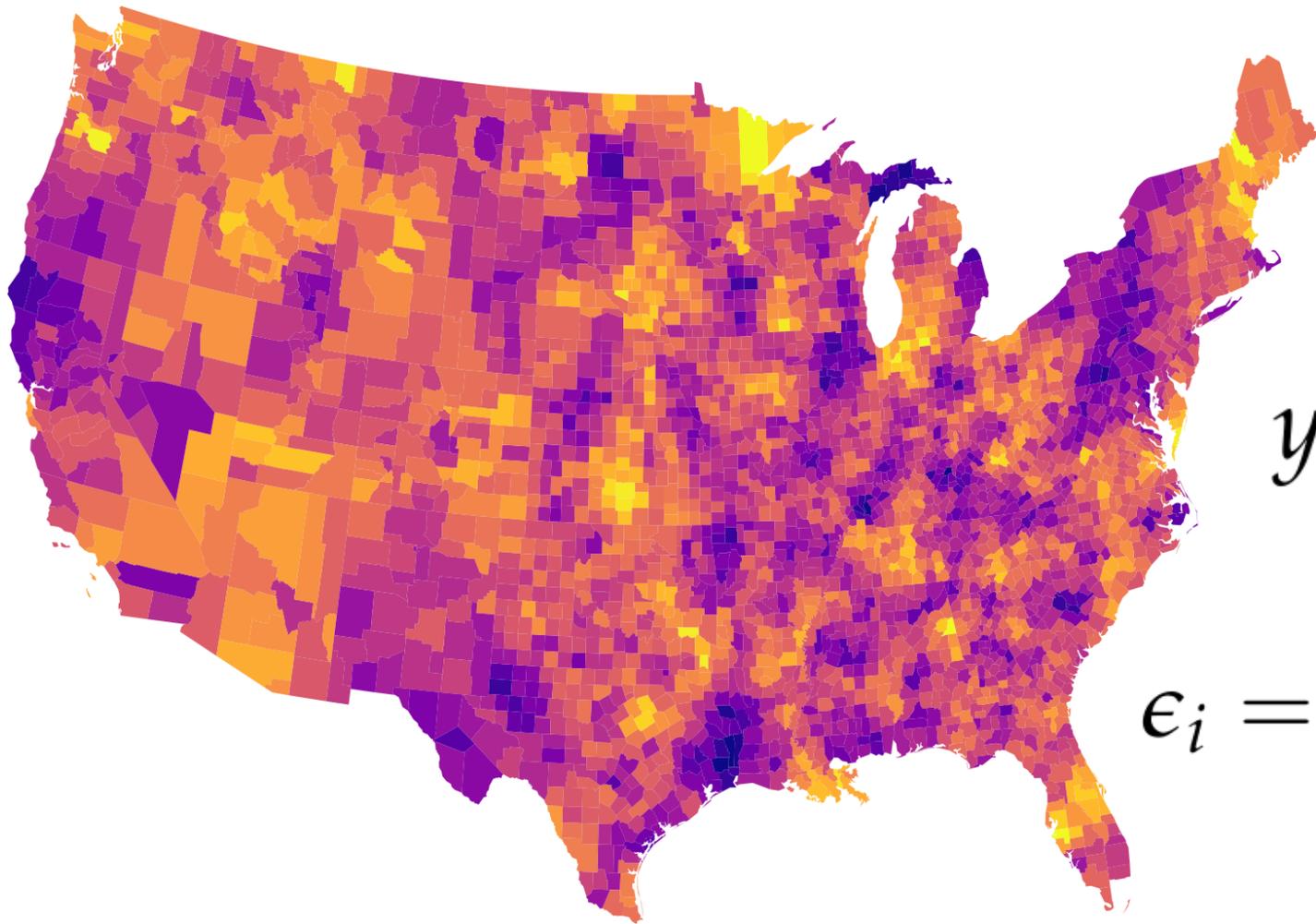


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# SAR MODELS

Still can have group-level model

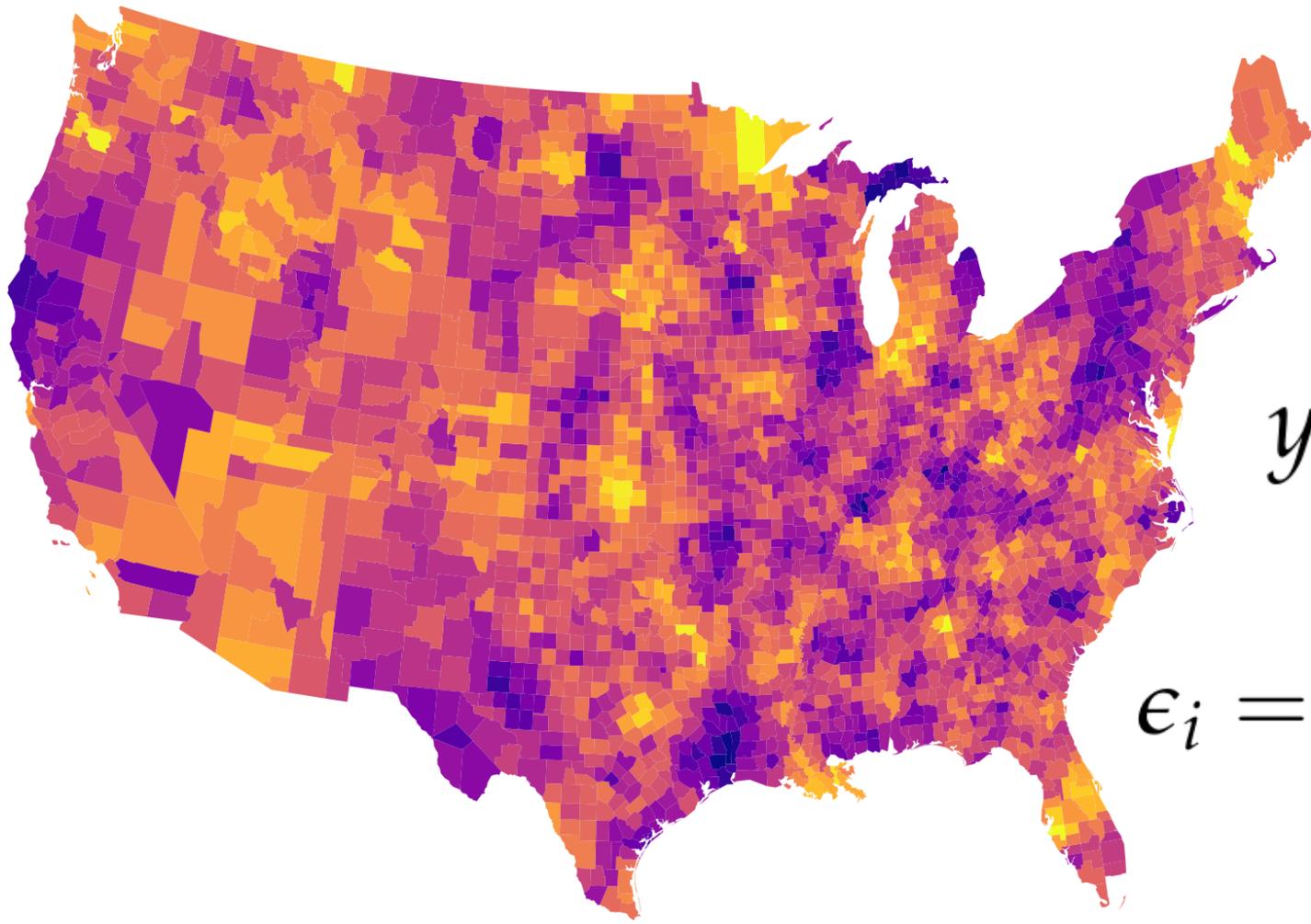


$$y_i = u_{j[i]} + \epsilon_i$$

$$\epsilon_i = \lambda \sum_i^N w_{ij} \epsilon_j + e_i$$

# SAR MODELS

Your surroundings matter. (Anselin 1988)



$$y_i = u_{j[i]} + \epsilon_i$$

$$\epsilon_i = \lambda \sum_i^N w_{ij} \epsilon_j + e_i$$

## SAR MODELS

This results in no change in estimates, but corrects overconfidence. **Estimates become less certain!**

$$y_i = u_{j[i]} + \epsilon_i$$

$$\epsilon_i = \lambda \sum_i^N w_{ij} \epsilon_j + e_i$$

$$u_j = a + \zeta_j$$

$$\zeta_j = \theta \sum_k^J m_{jk} \zeta_k + v_i$$

Responses center on group means

Response-level error is spatially smoothed

Groups center on grand mean

Region-level error is spatially smoothed

**SPATIAL  
MULTILEVEL**

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## DEPENDENCE THREE WAYS

RESPONSE SPATIAL

Nearby observations depend on one another, regardless of their group.

PLATIAL

Observations within groups depend on one another.

GROUP SPATIAL

Nearby groups depend on one another.

# SPATIAL MULTILEVEL

# THE GEOGRAPHIC DICHOTOMY

space & place are core to geography

## DEFINING A CLASSIC MODEL

multilevel models use place to estimate better

## IS PLACE JUST ABOUT GROUP?

space is ignored, and that's important

## **DOES IT REALLY MATTER?**

one bad nut can spoil the baklava

# SAR ERRORS

Used because they:

1. capture geographic similarity
2. correct artificially precise estimates
3. leave estimates values alone (in expectation)

# MULTILEVEL

Used because they:

1. capture w/in group similarity between observations
2. make group-level estimates more certain
3. shrink estimates for small groups with little info

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Used because they:

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# MULTILEVEL

Used because they:

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# INTRINSICALLY

# OPPOSED!

# On Spatial and Spatial Dependence: Examining Shrinkage in Spatially-Dependent Multilevel Models\*

Levi John Wolf<sup>1,2</sup>, Luc Anselin<sup>2</sup>, Daniel Arribas-Bel<sup>2,3</sup>, and Lee Mobley<sup>4</sup>

Final Review,

*Annals of Am. Assoc. Geog.*



[osf.io/ks6t3](https://osf.io/ks6t3)

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“Classic multilevel models may understate the uncertainty of the region-level parameter estimates and overstate their magnitude when spatial dependence exists at either level of the model.”

“Intuition from simpler models must be updated for more complex models ... ‘nuisance’ dependence in region-level variance components will show up as ‘substantive’ dependence.”

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$$\hat{u}_{j,mlm} = \frac{\sigma_u^2}{\sigma_u^2 + \frac{\sigma_e^2}{n_j}} \hat{u}_{j,ols}$$

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patial
spatial
spatial
patial

Because shrinkage is about variation, all of the (co)variation shows up!  
 Also can't separate out a single site anymore!

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MULTILEVEL**

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Groups center on grand mean

$$\zeta_j = \theta \sum_k^J m_{jk} \zeta_k + v_i$$

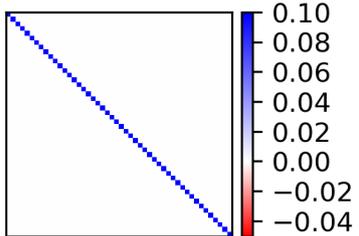
Region-level error is spatially smoothed

“Turn off” each process & see what happens.

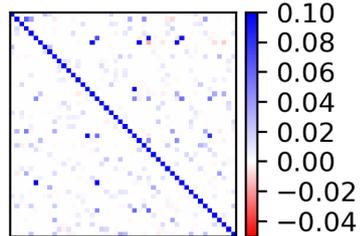
6 models in total. 2 single-level & 4 multilevel with varying configurations of regional & response-level spatial & platial dependence.

# SPATIAL MULTILEVEL

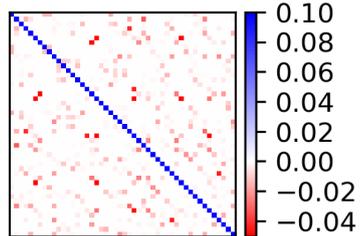
Standard MLM



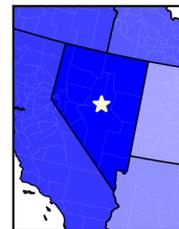
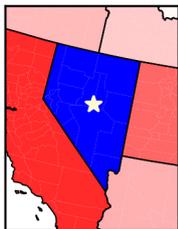
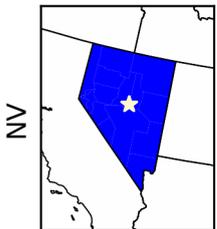
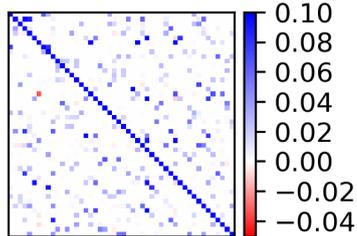
MLM Upper-SE



MLM Lower-SE



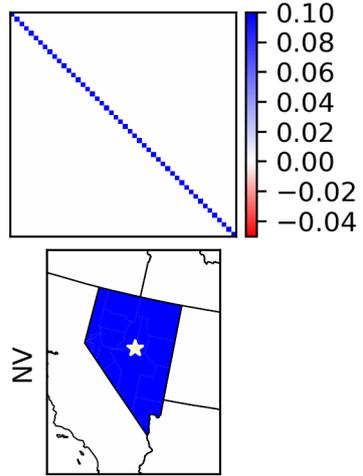
MLM Dual-SE



Because shrinkage is about variation, all of the (co)variation shows up!  
Also can't separate out a single site anymore!

“Intuition from simpler models must be updated for more complex models ...  
'nuisance' dependence in region-level variance components will show up as  
'substantive' dependence.”

Standard MLM



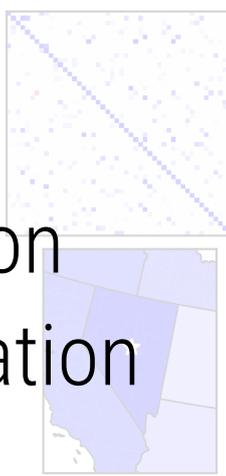
MLM Upper-SE



MLM Lower-SE



MLM Dual-SE

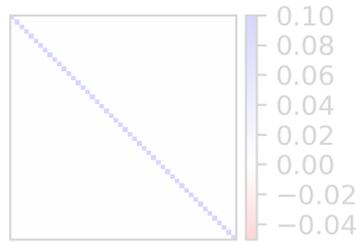


Depends on  
Group size  
Within group variation  
Between group variation

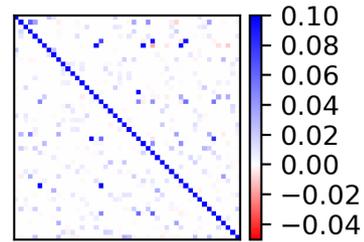
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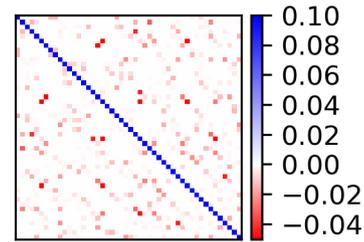
Standard MLM



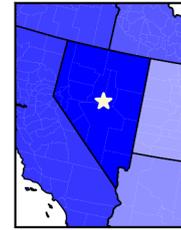
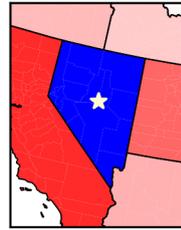
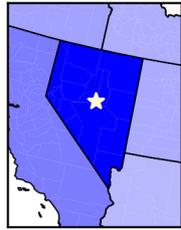
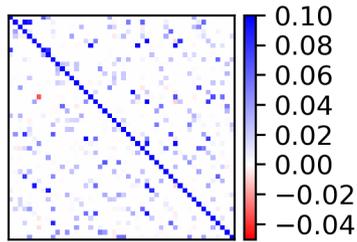
MLM Upper-SE



MLM Lower-SE



MLM Dual-SE

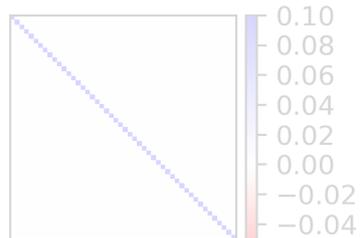


Cross state  
dependence at both  
levels can matter!

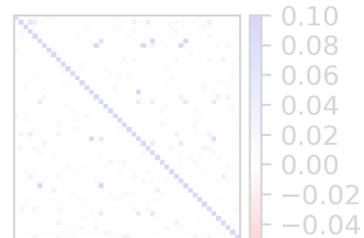
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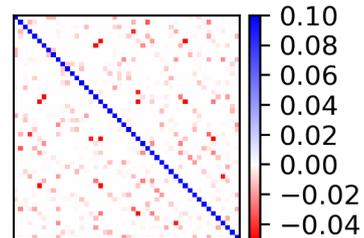
Standard MLM



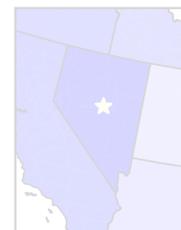
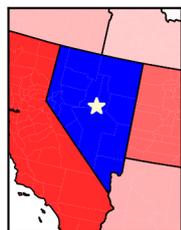
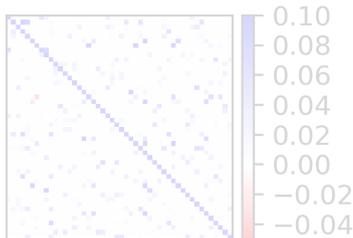
MLM Upper-SE



MLM Lower-SE



MLM Dual-SE



Even without group spatial dependence, information leaks between adjacent responses!

Because shrinkage is about variation, all of the (co)variation shows up!  
Also can't separate out a single site anymore!

“Intuition from simpler models must be updated for more complex models ...  
'nuisance' dependence in region-level variance components will show up as  
'substantive' dependence.”

# On Spatial and Spatial Dependence: Examining Shrinkage in Spatially-Dependent Multilevel Models\*

Levi John Wolf<sup>1,2</sup>, Luc Anselin<sup>2</sup>, Daniel Arribas-Bel<sup>2,3</sup>, and Lee Mobley<sup>4</sup>

Final Review,

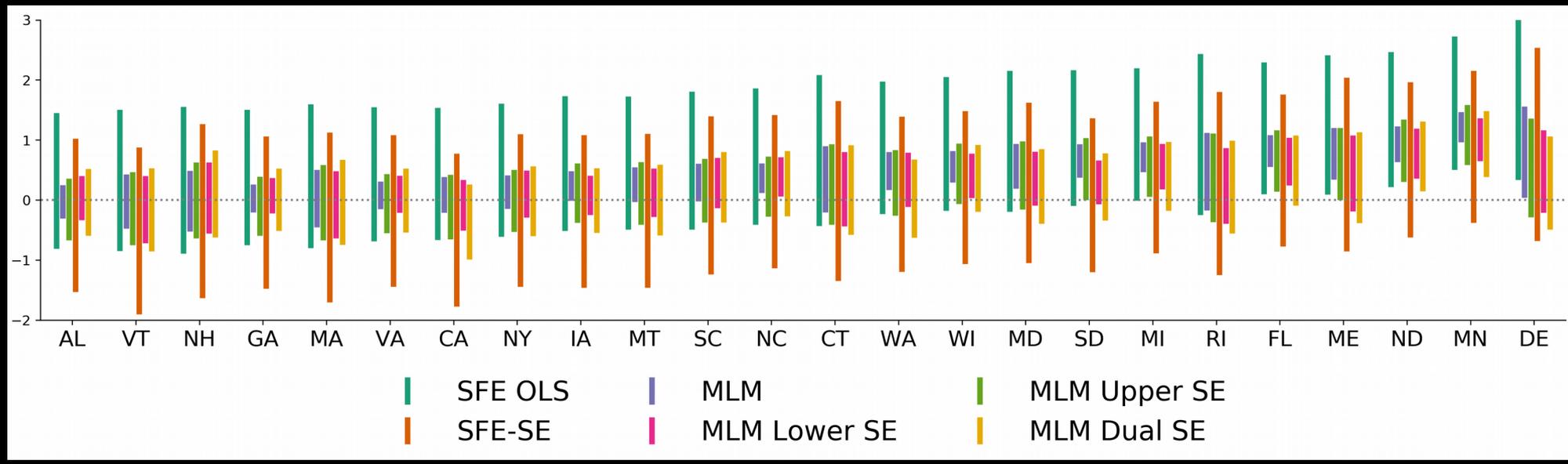
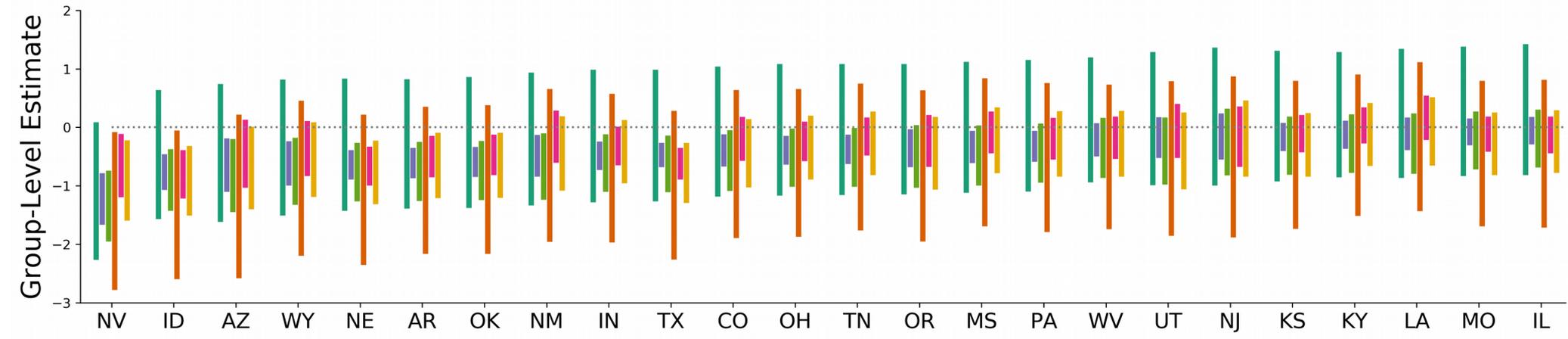
*Annals of Am. Assoc. Geog.*

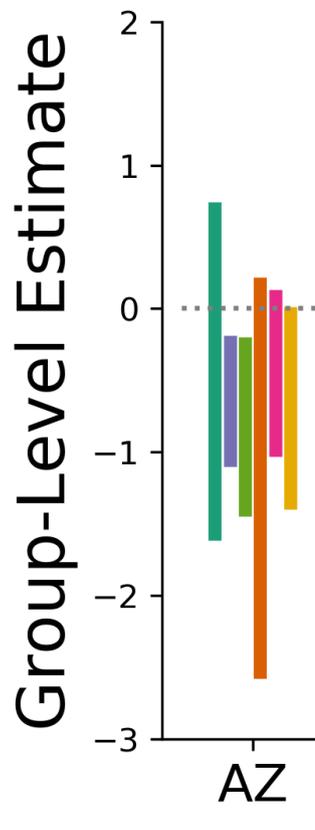


[osf.io/ks6t3](https://osf.io/ks6t3)

“Classic multilevel models may understate the uncertainty of the region-level parameter estimates and overstate their magnitude when spatial dependence exists at either level of the model.”

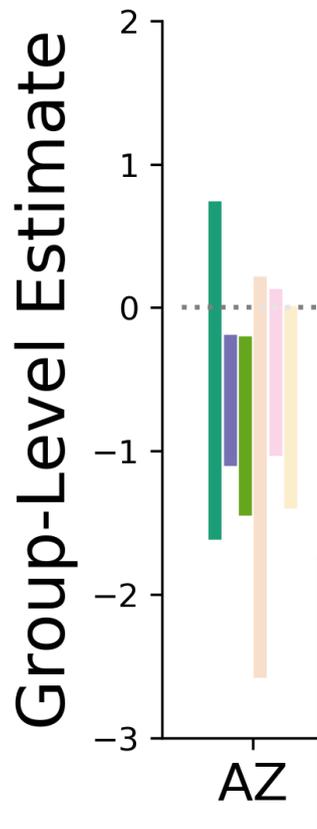
“Intuition from simpler models must be updated for more complex models ... ‘nuisance’ dependence in region-level variance components will show up as ‘substantive’ dependence.”





95% posterior density intervals for Arizona's intercept in a model predicting colorectal cancer uptake among medicare recipients after introduction of Medicare Fee-For-Service

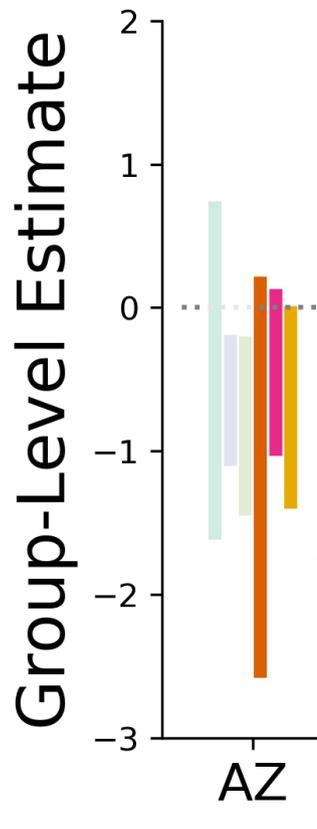
“Classic multilevel models may understate the uncertainty of the region-level parameter estimates and overstate their magnitude when spatial dependence exists at either level of the model.”



## Cool Tone Colors

no response spatial dependence

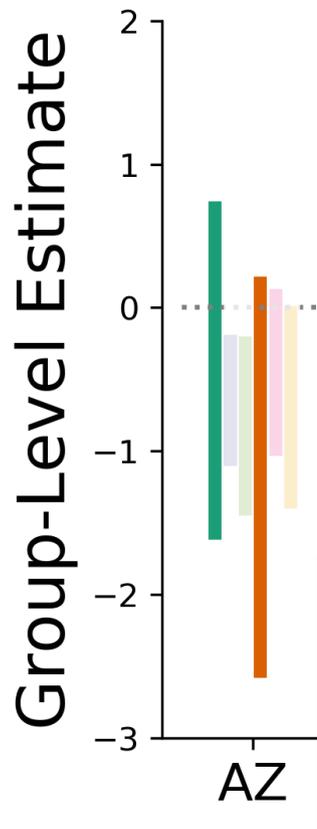
“Classic multilevel models may understate the uncertainty of the region-level parameter estimates and overstate their magnitude when spatial dependence exists at either level of the model.”



## Warm Tone Colors

response spatial dependence  
(possibly in addition to region)

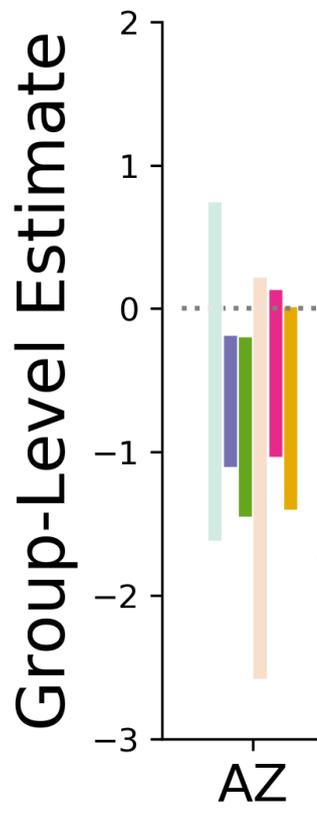
“Classic multilevel models may understate the uncertainty of the region-level parameter estimates and overstate their magnitude when spatial dependence exists at either level of the model.”



## Long Bars

Fixed effect estimates using  
maximum likelihood

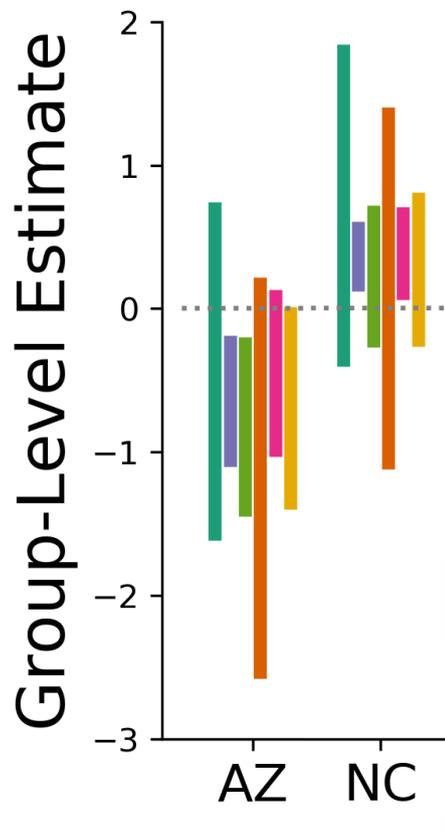
“Classic multilevel models may understate the uncertainty of the region-level parameter estimates and overstate their magnitude when spatial dependence exists at either level of the model.”



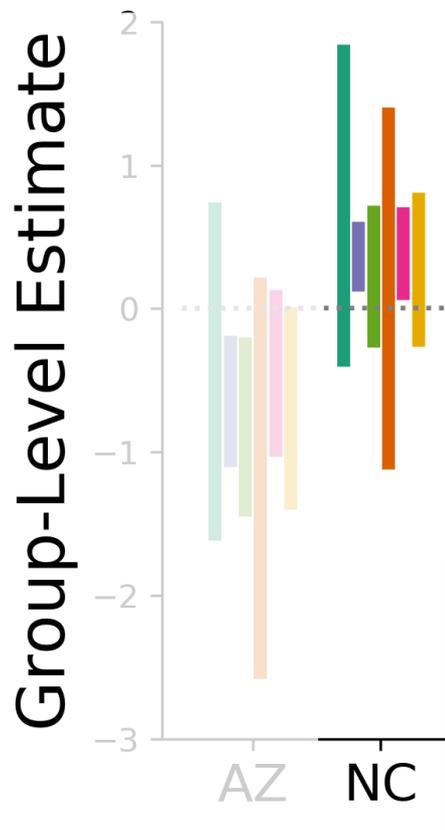
## Short Bars

Multilevel estimates using  
Bayesian Gibbs Sampling

“Classic multilevel models may understate the uncertainty of the region-level parameter estimates and overstate their magnitude when spatial dependence exists at either level of the model.”



“Classic multilevel models may understate the uncertainty of the region-level parameter estimates and overstate their magnitude when spatial dependence exists at either level of the model.”

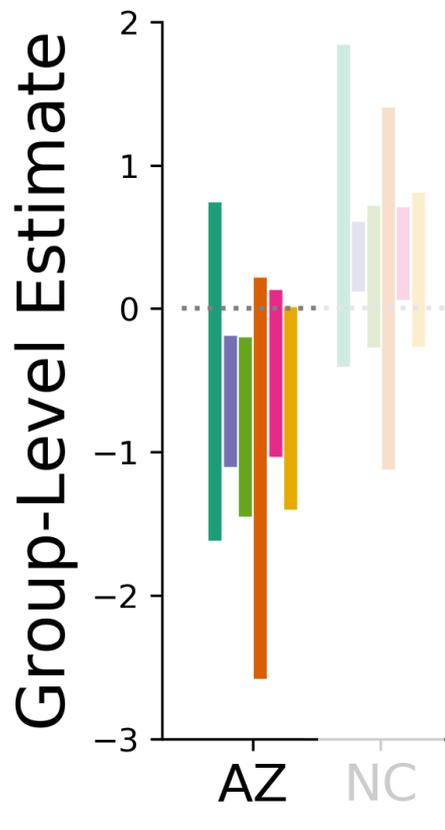


All multilevel models improve in certainty.

But, models with regional spatial dependence have weaker improvements!

The classic multilevel model is overconfident!

“Classic multilevel models may **understate the uncertainty** of the region-level parameter estimates and overstate their magnitude when spatial dependence exists at either level of the model.”



All multilevel model estimates shrink.

But ones that allow for response-level spatial dependence do so dramatically more!

The classic multilevel model is exaggerated!

“Classic multilevel models may understate the uncertainty of the region-level parameter estimates and **overstate their magnitude** when spatial dependence exists at either level of the model.”

**THE GEOGRAPHIC DICHOTOMY**

**DEFINING A CLASSIC MODEL**

**IS PLACE JUST ABOUT GROUP?**

**DOES IT REALLY MATTER?**

## THE GEOGRAPHIC DICHOTOMY

space & place are core to geography

## DEFINING A CLASSIC MODEL

multilevel models use place to estimate better

## IS PLACE JUST ABOUT GROUP?

they ignore space, and that's important

## DOES IT REALLY MATTER?

they can be overconfident and exaggerated

# PLACE & SPACE IN MULTILEVEL MODELS

TOWARDS MORE USEFUL GEOGRAPHY  
IN MULTILEVEL MODELS



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